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1855

STRENGTH OF MATERIALS

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INDUSTRIAL SERIES

STRENGTH OF MATERIALS

BY

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Prepared under the Direction
of the Division of Engineering Extension
THE PENNSYLVANIA STATE COLLEGE

McGRAW-HILL BOOK COMPANY, Inc.
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1941

STRENGTH OF MATERIALS

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PREFACE

This text on Strength of Materials has been written to meet fully the needs of the field of industrial training by providing an adequately simplified discussion of the principles involved. It is also well adapted for those who desire to study at home through correspondence instruction and was given a thorough test in the fields already mentioned before being printed in book form.

The illustrative numerical examples which are included have been solved without the omission of any step, which greatly adds to their value for study purposes. The problems have been taken from the many practical fields of engineering and design and incorporate the fundamental principles of both mechanics and strength of materials. Mathematics above the level of trigonometry has been entirely avoided in presenting the theory and practice involved.

The subject includes the usual topics of tension and compression; riveted joints are explained in an up-to-date manner, and welded connections are included. Beam design is thoroughly discussed and beam deflections are considered. The design of steel and timber columns is covered by the latest methods and equations of design.

The author is indebted to J. M. Holme for reading the text and for his helpful suggestions. He also desires to thank the Carnegie-Illinois Steel Corporation for permission to reproduce certain tables from the *Carnegie Pocket Companion*, as found in the appendix.

J. W. BRENEMAN.

THE PENNSYLVANIA STATE COLLEGE,
June, 1941.

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FOREWORD

This volume is one of a series of texts prepared by members of the staff of the School of Engineering of The Pennsylvania State College. While these books may be of value in college teaching and elsewhere, their principal purpose is for use in adult teaching.

Experience in extension service indicates that there is a great lack of teaching materials suitable for the instruction of adult classes. Such extension classes at The Pennsylvania State College are composed of individuals who wish to study material which has fairly direct application. This is particularly true of students who are working in industry and who carry on part-time study, as well as others who are preparing for jobs in industry. A common school education may make up the entire formal schooling of many of these students. Such students qualify by reason of self-education in many cases and an experience in industry which constitutes a valuable apprenticeship as well.

Extension instructors are selected on the basis of their practical experience in a particular field as well as on the basis of their academic preparation. It follows, therefore, that text materials used in such classes should be readily readable, understandable, and "practical."

The author of this volume on Strength of Materials, J. W. Breneman, Associate Professor of Engineering Mechanics in the School of Engineering, has incorporated in this volume some of the results of a wide experience in the fields of industrial engineering training as well as in teaching at the college level. Altogether, three books, covering Mechanics, Strength of Materials, and Mathematics, have been prepared and used in temporary form for several years before being put into their final printed edition. It is hoped that this book plays an important part in practical industrial training programs in progress throughout this country.

This Industrial Series will include texts on mathematics, blue print reading, engineering drawing, mechanics, strength of materials, machine design, electricity, and others. While a modicum of theory will be included, stress will be laid on the application of principles of these subjects to important practical problems that are common in industry.

E. L. KELLER, DIRECTOR,
DEPARTMENT OF ENGINEERING EXTENSION.

THE PENNSYLVANIA STATE COLLEGE,
June, 1941.

STRENGTH OF MATERIALS

CHAPTER I

SIMPLE STRESSES

Stress and Strain.—When a force or load is applied to a piece of material, the size and shape of the material is changed slightly. This change of size or shape is called strain or deformation.

Stress is a name given to the forces which are acting. The external forces cause reactions to be set up within the material to withstand them. Both the reactions, or resisting forces, and the external forces are called stresses.

When the *external* or the applied forces *exceed* the *interna'* resisting stresses, the material will *rupture* or *break*.

The terms *stress* and *strain* are generally defined together: stress is the combination of all forces which are acting on a body, while strain is the effect on the body.

Simple Stresses.—There are three kinds of simple stresses and they are known by the names *tension*, *compression*, and *shear*.

Tension, or a tensile stress, is one caused by a force which is tending to pull out or stretch a piece. The cables on a crane are subjected to a tensile stress when a load is being lifted.

Compression, or a compressive stress, is one caused by a force tending to push together or shorten a piece. The base of a heavy machine is in compression for the entire weight of the machine is bearing down and tending to crush the base.

Shear, or a shearing stress, is one caused by a force which tends to cause one part of a piece to slide over another part. Punching holes in boiler plate or cutting any sheet metal is an example of shear.

Figures 1, 2, and 3 show how the forces act in these simple stresses. In Fig. 1 the forces are tending to pull the piece apart so that it is in tension. In Fig. 2 the forces are tending to crush the piece so that it is in compression. In Fig. 3 some of the forces are tending to move the metal one way while the other forces are tending to move it in the opposite direction. The two parts of the piece, therefore, are tending to slide over each other or to shear at the middle as shown by the dotted line. This action at Fig. 3 is just the same as if a pair of shears were cutting the material.

In Figs. 1 and 2, the forces which are acting on the bodies are perpendicular to the cross-section area. They are applied so that the line of action of the force coincides with the axis of the body. In Fig. 3, the forces are applied parallel to the cross-section area, causing the shearing action.

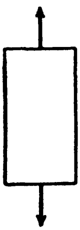


Fig. 1.

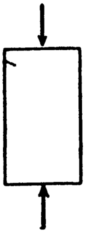


Fig. 2.

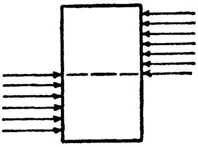


Fig. 3.

Loads.—The external forces which set up the stresses in a body are called *loads*. The load on a bridge, therefore, will be the weight of its own members plus the weight of anything passing over it. The common form of loads which act upon a piece of material are the *dead loads* and the *live* or *variable loads*.

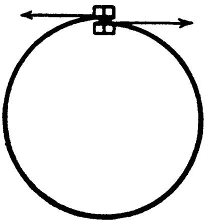


Fig. 4.

Dead Load.—A dead load is one which always remains the same and, therefore, is one which causes a permanent and unchangeable stress in the piece it is acting upon. The weight of a building on the foundations will be dead load in compression. Wires stretching between poles have a dead load in tension. The rivets in a tank will have a dead load in shear if the pressure in the tank always remains the same. Figure 4 shows why a rivet will always have a shearing action. The pressure in the cylinder tends to flatten out the plate so that the pull on the rivet is as shown by the arrows. This pull tends to shear the rivet off between the plates.

Live Load.—A live load is one which is frequently changing or one which is alternately applied and removed. The load on a crane is variable due to the different weights being lifted.

Unit Stresses.—Unit stresses due to axial loads are found as follows: Fig. 5 shows a straight piece of material subjected to a load P causing a resisting stress to be set up on any cross-section perpendicular to the axis of the piece. As action and reaction are always equal and opposite, it follows that (isolating the part of the piece shown in Fig. 6) the total resisting stress is equal to the product of the area of the cross-section A and the unit stress S, and hence

$$\text{Load} = \text{resisting stress}$$
$$P = AS$$

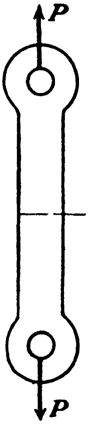


FIG. 5.

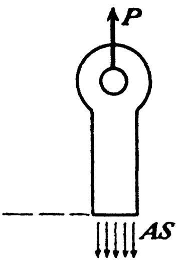


FIG. 6.

Unit stresses should always be given in pounds per square inch. Such stresses are called unit stresses and will be found by dividing the total stress in an object by the square inches in the cross-section area. Hence, if the total stress on a bar is 30,000 lb. and it has a cross section area of 2 sq. in., then the unit stress will be 30,000 divided by 2 or 15,000 lb. *per sq. in.*

The **ultimate strength** of a material is the maximum unit stress which the material can resist just before it breaks.

Factor of Safety.—In designing a part for a machine or for a structure, the total amount of the load on the piece is first calculated. The piece is then made large enough so that it will withstand this total load without being subjected to a stress which will bring about a permanent deformation. *Hence the allowable stress will always be much less than the ultimate strength of the piece.* If the stress on the piece is only one half the ultimate strength it may be said that the piece is only doubly safe, or if the stress is only one third of the ultimate strength, the piece is three times safe. *This relation of the actual stress in the piece and the ultimate strength of it is called the factor of safety.* Hence, if a chain has an ultimate strength of 50,000 lb. and it is to carry a maximum load of 10,000 lb., then it is five times safe or its *factor of safety is* 5.

Since the factor of safety is based on the ultimate strength of the material, it follows that the ultimate strength and the factor of safety for all materials under all conditions must be known.

These have been found by experiment and tested by long experience. Table 1 shows the ultimate strength of various materials. When some value is omitted from the table it means that the material is not used for that kind of stress. For example, brick and stone are used for compression only as in foundations, so that no values are given for tension and shear.

TABLE 1.—ULTIMATE STRENGTH IN POUNDS PER SQUARE INCH

Material	Tension	Compression	Shear
Cast iron	......	85,000	......
Wrought iron	50,000	50,000	35,000
Steel castings	65,000	65,000	56,000
Soft steel	55,000	55,000	45,000
Machine steel	64,000	64,000	50,000
Structural steel	64,000	64,000	50,000
Wood	10,000	8,000	3,000
Brick	......	3,000	......
Stone	......	6,000	......

Table 2 shows the factors of safety which may be used for different classes of work. It will be noticed that materials which are uniform in structure, such as iron and steel, have a smaller factor of safety than materials which are not uniform in structure.

TABLE 2.—FACTORS OF SAFETY

Material	Dead load	Variable load	Members subject to sudden shocks
Cast iron	5	8	15
Wrought iron	3	4½	10
Steel	4	6	10
Timber	10	14	18
Brick and stone	15	30	

The values in Table 2 are average ones and can be made larger or smaller depending upon the conditions. For example, if the load on a member is only slightly variable, then a factor of safety between the values for dead load and variable load, may be used.

Both of these tables are of the utmost importance and should be studied carefully. The student should soon have all the values firmly fixed in mind so that problems can be solved without referring to them.

The values in Table 1 were all expressed in pounds per square inch so that these must always be multiplied by the area of the piece to find the ultimate strength of that piece.

Example.—Find the ultimate strength and the allowable stress for a bar of structural steel $2\frac{1}{2}$ in. by $1\frac{1}{2}$ in., if it is to be used in tension, and for a variable load.

Solution.—The cross section area of this bar will be

$$2\tfrac{1}{2} \times 1\tfrac{1}{2} = 3\tfrac{3}{4} \text{ sq. in.}$$

Table 1 shows that structural steel has an ultimate strength in tension of 64,000 lb. per sq. in. Hence the ultimate strength of this bar will be

$$3\tfrac{3}{4} \times 64{,}000 = 240{,}000 \text{ lb.}$$

From Table 2 we find that the factor of safety for steel with a variable load is 6. Hence the allowable stress will be

$$\frac{240{,}000}{6} = 40{,}000 \text{ lb. } \textit{Answer}$$

Example.—A wrought iron bar $\frac{1}{2}$ in. in diameter is withstanding a load of 3,000 lb. in compression. Calculate the unit stress.

Solution.—The cross section area of a $\frac{1}{2}$-in. bar will be

$$\frac{\pi(\frac{1}{2})^2}{4} = 0.196 \text{ sq. in.}$$

hence,

$$\frac{3{,}000}{0.196} = 15{,}000 \text{ lb. per sq. in. } \textit{Answer}$$

In the same manner the size of a member can be determined if the load is known. For example, suppose a piece of structural steel must be designed so that it will hold a load of 50,000 lb. in tension and as a dead load.

From Table 1 we find that structural steel has an ultimate strength of 64,000 lb. in tension, and from Table 2 we find that the factor of safety for structural steel as a dead load is 4. Hence, the allowable load per square inch will be

$$\frac{64{,}000}{4} = 16{,}000 \text{ lb. per sq. in.}$$

Now if 1 sq. in. will safely hold 16,000 lb., then to find the number of square inches to hold 50,000 lb. we divide 50,000 by 16,000, or

$$\frac{50{,}000}{16{,}000} = 3.13 \text{ sq. in. required}$$

If a square bar is desired it will now be necessary to find the side of a square which has 3.13 sq. in. for the area. This is done by extracting the square root of the area, or

$$\sqrt{3.13} = 1.77 \text{ in.}$$

Hence, if the bar is square it will be 1.77 in. on each side.

If the bar is to be circular instead of square, then we must find the diameter of a circle whose area is 3.13 sq. in.

$$\text{Area of circle} = \frac{\pi D^2}{4}$$

Hence

$$3.13 = \frac{3.1416 D^2}{4}$$

from which

$$D^2 = 3.98 \qquad \text{or} \qquad D = 1.99 \text{ in. or } 2 \text{ in. } \textit{Answer}$$

The following examples have been worked out so that the student may get a clear idea as to the design of pieces when subjected to simple stresses. These examples should be carefully studied and understood thoroughly before proceeding with the study of the text.

Example.—The steel hoisting chain in a crane has a cross section area of $\frac{7}{8}$ sq. in. Is it safe for this crane to lift a load of 6 tons?

Solution.—The chain will be in tension and Table 1 shows that the ultimate strength of steel in tension is 64,000 lb. per sq. in. The load on a crane is variable so that from Table 2 the factor of safety should be 6. Hence, the allowable load will be

$$\frac{64{,}000}{6} = 10{,}666 \text{ lb. per sq. in}$$

Now since the cross section area of the crane is $\frac{7}{8}$ sq. in., the allowable load on the chain will be

$$\frac{7}{8} \times 10{,}666 = 9{,}330 \text{ lb.}$$

Therefore the maximum load this crane should lift is 9,330 lb. so that it will not be safe for it to lift 6 tons which is 12,000 lb.

Example.—A steel tie rod on a bridge must be made to withstand a pull of 5,000 lb. Find the diameter of the rod assuming a factor of safety of 5.

Solution.—The allowable stress will be the ultimate strength divided by the factor of safety, or

$$\frac{64{,}000}{5} = 12{,}800 \text{ lb. per sq. in.}$$

Hence,

$$\frac{5{,}000}{12{,}800} = 0.39 \text{ sq. in. required}$$

from which

$$0.39 = \frac{\pi D^2}{4} \qquad \text{or} \qquad D = 0.71 \text{ or about } \frac{11}{16} \text{ in. } \textit{Answer}$$

Example.—What must be the size of a square cast iron block to support a weight of 60 tons dead load?

Solution.—From the tables the ultimate strength and the factor of safety are found to be 85,000 lb. per sq. in. and 5 respectively.

Hence,

$$\frac{85{,}000}{5} = 17{,}000 \text{ lb. per sq. in. allowable stress}$$

The total load is 60 tons or 60 × 2,000 = 120,000 lb., hence,

$$\frac{120{,}000}{17{,}000} = 7 \text{ sq. in. required}$$

Since the block is square it follows that

$$\sqrt{7} = 2.7 \text{ or } 2\tfrac{3}{4} \text{ in. } \textit{Answer}$$

Example.—A threaded bolt is shown in Fig. 7. When the bolt is tightened by screwing up the nut, a tensile force is applied which slightly stretches the bolt. If a sufficient tensile load P is applied to the bolt, it may break, as shown in Fig. 8, across the area of a circle whose diameter is the net (root) diameter of the bolt. Thus a $\frac{3}{4}$-in.-diameter bolt, as shown in Fig. 8, is stretched by tightening the nut until the bolt breaks, when the total tensile force is 18,700 lb. What is the ultimate unit tensile stress of the material?

From the Appendix, the net diameter c of a ¾-in.-bolt is 0.620 in. and the net area (root area) is 0.302 sq. in. Then

$$S = \frac{P}{A} \quad \text{and} \quad S = \frac{18{,}700}{0.302} = 61{,}920 \text{ lb. per sq. in.}$$

Note: Failure of the bolt may occur by a stripping of the threads, rather than by a rupture of the root area. This failure will be given consideration under the question of shear.

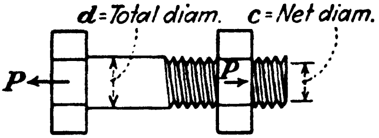


Fig. 7.

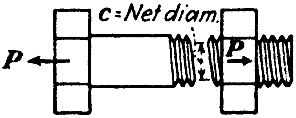


Fig. 8.

Example.—The piston in an engine is 10 in. in diameter and the steam pressure is 100 lb. per sq. in. Find the proper diameter of the machine steel piston rod.

Solution.—The area of the piston will be

$$\text{Area} = \frac{\pi D^2}{4} = \frac{3.14 \times 10 \times 10}{4} = 78.54 \text{ sq. in.}$$

Since the pressure is 100 lb. per sq. in., the total pressure will be

$$78.54 \times 100 = 7{,}854 \text{ lb.}$$

From the tables we find that the ultimate strength is 64,000 lb. per sq. in. and we will take a factor of safety of 10 since the load comes very suddenly. Hence the allowable stress will be

$$\frac{64{,}000}{10} = 6{,}400 \text{ lb. per sq. in.}$$

Now if the total load on the piston rod is 7,854 lb., and the allowable stress is 6,400 lb. per sq. in., then the area must be

$$\frac{7{,}854}{6{,}400} = 1.23 \text{ sq. in., area of rod}$$

Hence, to find the diameter

$$1.23 = \frac{\pi D^2}{4} \text{ from which } D = 1.25 \text{ in. } \textit{Answer}$$

Problems

1. Name and define in your own words the three kinds of simple stresses.

2. Give original examples of each kind of simple stress.

3. Explain what is meant by "factor of safety."

4. Name and define in your own terms the most common form of "loads."

5. Define and give an example of "ultimate stress."

6. A round iron bar $\frac{1}{2}$ in. in diameter sustains a pull of 4000 lb. What is the unit stress on the bar?

7. A structural steel bar is to withstand a suddenly applied load of 24,000 lb. Design a round and a square bar which will be safe.

8. How great a weight can be safely put on a 12-in. by 12-in. timber?

9. Figure 9 shows a column resting upon a stone base. If the weight of this column is 18,000 lb., what is the cross section area of the stone base?

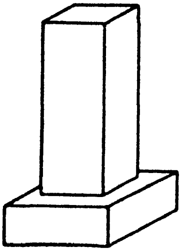


FIG. 9.

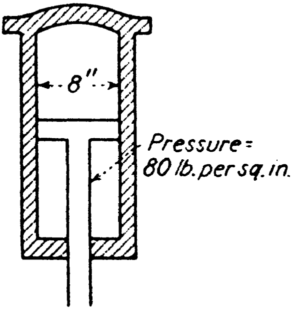


FIG. 10.

10. Figure 10 shows a section of an air hoist with a piston 8 in. in diameter and a piston rod 1 in. in diameter. If the air pressure is 80 lb. per sq. in. find the maximum load that can be lifted safely and determine the safety of the steel piston rod.

11. A $1\frac{1}{2}$-in. diameter bolt failed in tension when the unit stress was 59,500 lb. per sq. in. What was the total tensile force causing the bolt to fail?

Elasticity.—When a rubber band is stretched out and let go, it will go back with a quick snap. Thus, we speak of rubber as being elastic. Many people think of the stretching as being the measurement of its elastic qualities, but, as a matter of fact, it is the force and amount of the return which measures the amount of elasticity. *The elasticity of a body, therefore, is its tendency to return to its original size and shape after being deformed.*

When a steel spring has been wound and let go it will return to its original position with a great force which shows that steel is highly elastic. The shooting of an arrow from a bow is due to the elasticity of the wood, for when the bow is bent and then released it straightens again with considerable force.

Thus, it is seen that most materials of construction may be considered as perfectly elastic for small stresses. That is, when small stresses are set up in these bodies any small deformations which occur will disappear when the stress is removed.

Effect of Stresses.—*In elastic bodies such as iron or steel the deformation resulting from a small stress will be in direct proportion to the stress which causes it.* This means that if a body is subjected to a small stress and then this stress is doubled, any deformation which resulted from the first small stress will also be doubled under the doubled stress. If the stress is again doubled then the deformation will be doubled again.

However, if these stresses are continually made greater a stress will soon be reached where this relation will not hold true. That is, a stress will be reached where the deformation will be more than doubled when the stress is doubled. And at this stress it will also be observed that when the stress is removed, the body will *not* go back entirely to its original size and shape. In other words, a part of the deformation has become permanent.

Elastic Limit.—The *elastic limit* is defined as the greatest *stress* a material is capable of developing without a *permanent deformation* remaining upon *complete removal* of the *load.* It is also the load or stress which ceases to be proportional to the deformation. Or it may be said that the elastic limit is that stress where a permanent deformation will begin. An example will illustrate this point. A bar of metal 1 sq. in. in area, when subjected to a tensile stress acted as follows: when the tension was 2500 lb. the deformation, or in this case the elongation or stretch, was found to be 0.01 per cent of its original length. When the tension was doubled to 5000 lb., the elongation was found to be doubled to 0.02 per cent of the original length. For each additional increase in tension of 2500 lb. the elongation increased another 0.01 per cent, and this continued until the tension was 25,000 lb. Hence, up to this point, the elongation was in proportion to the stress. However, when the next 2500 lb. was added making the total 27,500 lb. tension, it was found that the increase in elongation was *not* an additional 0.01 per cent, but rather almost 0.02 per cent. Hence, for this bar the elastic limit was 25,000 lb., since any tension under this amount caused an elongation in proportion to 25,000 lb.

Hooke's Law.—It follows that when a material is placed under stress, the unit stress in the material is proportional to the unit strain at any point provided that the elastic limit has not been exceeded. That is, for stresses within the elastic limit, all elastic materials behave according to the same law, known as Hooke's law. It can be expressed mathematically:
let

Δ = total deformation (strain)
L = original length

then

$$\text{Unit deformation or unit strain } e = \frac{\Delta}{L}$$

On page 3 it was shown that unit stress $= \frac{P}{A} = S$; then the ratio of $\frac{\text{unit stress}}{\text{unit strain}} = \frac{S}{e} =$ a constant. This constant is called the modulus of elasticity. The numerical value of the ratio is constant for any one material whenever subjected to the same kind of stress. Hence, letting the modulus of elasticity $= E$, the constant ratio is $E = \frac{S}{e}$.

The modulus of elasticity varies for the material and the type of stress in the material. Table 3 shows average values of the modulus of elasticity of some materials.

TABLE 3.—MODULUS OF ELASTICITY
(Pounds per square inch)

Material	Type of stress	
	Tension or compression	Shear
Cast iron	12,000,000	6,000,000
Wrought iron	28,000,000	10,000,000
Steel	29,000,000	12,000,000
Steel alloys	29,000,000	12,000,000
Woods	800,000 to 1,800,000	

The elastic limit of materials is found by actually testing a piece in the same manner that the bar in the preceding paragraph was tested. The bar is put in a testing machine and various loads applied and the stretch measured. Thus the value of the elastic limit may be found accurately.

Ultimate and Breaking Stresses.—Various materials, when tested in tension or compression, give different results.

Referring to the piece of steel tested in tension in the above paragraph, when the stretching action is continued, the load required increases gradually but *not uniformly* until a *maximum* or *ultimate* value is obtained, after which it *decreases* until, with continued stretching, the steel breaks. The load at the time the piece breaks is called the *breaking load.*

These terms are illustrated in Fig. 11 which is a graph showing how a certain bar of steel acted when it was stretched until it broke. The vertical scale shows the amount of tension on the bar and the horizontal scale shows the corresponding elongation due to the stress.

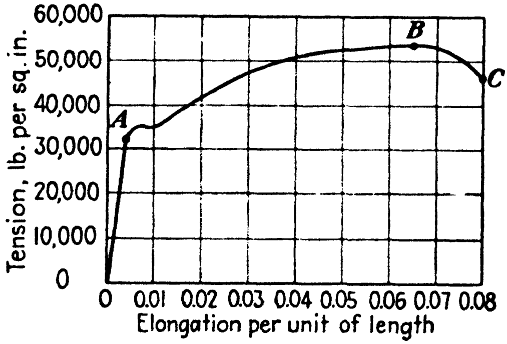


FIG. 11.

Up to the point A the graph is a straight line which shows that the elongation in the bar was directly proportional to the stress. After the point A, however, it will be seen that the rate of elongation increased. The point A, therefore, was the elastic limit which for this bar was 34,000 lb. per sq. in.

As more tension was applied the bar stretched out very rapidly until the point B was reached. It will be noticed that this point B is the highest one on the graph since, after it, the bar kept pulling out with a decreased amount of tension. The point B, therefore, is the ultimate strength of the bar, which in this case is 55,000 lb. per sq. in.

Point C on the graph illustrates the breaking point which is about 42,000 lb. per sq. in.

A piece of wood or cast iron tested in compression will have the same ultimate and breaking loads, since, when the maximum load is reached, the piece will break.

As an example of Hooke's law:

A tie rod is made of structural steel for which the modulus of elasticity is 29,000,000 lb. per sq. in. The rod is 12 ft. long and is 1 in. by 2 in. in cross section. When a load of 36,000 lb. is applied to the rod, how much does it stretch?

From Hooke's law, $E = S/e$ but $S = P/A$ and $e = \Delta/L$, then

$$E = \frac{\frac{P}{A}}{\frac{\Delta}{L}} = \frac{PL}{A\Delta}$$

where P = total load in pounds
L = *original length* in inches
A = *original area* in square inches
Δ = total deformation (elongation or contraction) in inches
E = modulus of elasticity in pounds per square inch

This equation is important as it shows the relation between the applied load and the total deformation.

From

$$E = \frac{PL}{A\Delta}, \qquad \Delta = \frac{PL}{AE}$$

Then, substituting the given values,

$$\Delta = \frac{36{,}000 \times 144}{2 \times 29{,}000{,}000} = 0.0894 \text{ in. total stretch}$$

and the unit stretch (stretch per inch of length) is

$$e = \frac{\Delta}{L} = \frac{0.0894}{144} = 0.00062 \text{ in. per in.}$$

Temperature Effects.—The expansion or contraction of materials due to a change in temperature induces stresses in the piece of material under consideration. Where the change in temperature is large, the stress set up is often large enough to cause a permanent deformation to be placed in the material. In some cases, failures have occurred from this cause as well as from ordinary metals not being as strong at exceedingly high or low temperatures as they are at normal temperatures.

The coefficient of linear expansion (or contraction) is the linear deformation per inch of length per degree change in temperature. Medium steel has a coefficient of linear expansion of 0.0000067 in. per in. per degree change in temperature Fahrenheit. See the Appendix for the coefficients of expansion of different metals.

From the definition of the coefficient of expansion the total deformation caused by a temperature change is determined as follows:

let

n = coefficient of linear expansion
t = change in temperature in degrees Fahrenheit

then

Δ = coefficient × change in temperature × length
$\Delta = ntL$ = total deformation due to temperature change

and

$$E = \frac{PL}{A\Delta}$$

hence

$$E = \frac{PL}{AntL} = \frac{P}{Ant}$$

from which $P = EAnt$.

Thus the total load induced by a temperature change is dependent on the modulus of elasticity, the area, the coefficient of linear expansion and the temperature change.

Example.—A medium steel tie rod 1 in. × 2 in. × 12 ft. long has each end fastened in a wall in such a manner that no slipping can take place. If the temperature drops from 110°F. to 20°F., what tensile load is placed in the rod?

E = 29,000,000 lb. per sq. in.
A = 1 in. × 2 in. = 2 sq. in.
n = 0.0000067 (from the Appendix, Table of Linear Expansion)
$t = 110° - 20° = 90°$
$P = EAnt$
$= 29{,}000{,}000 \times 2 \times 0.0000067 \times 90$
= 34,970 lb.

It is very essential that the conditions, under which the piece of material is placed, be understood as temperature changes will often change a stress from tension to compression or from compression to tension. Thus a short bar of soft steel is placed as a support and welded as shown in Fig. 12. The supports to which the bar is welded are fixed so that the 2 ft. distance between them is constant. The bar is 4 × 4 in. in cross section and is holding a *compressive* load of 192,000 lb. The temperature of the bar changes from 100°F. to 20°F. What is the final total load in the bar?

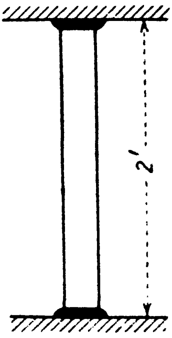


Fig. 12.

Initial load = 192,000 lb. compression

When the temperature decreases, the length of the piece must remain as 2 ft., hence the bar stretches an amount equal to the contraction caused by the decrease in temperature.

Since

$$P = EAnt$$

where

$n = 0.0000061$ for soft steel

$P = 29{,}000{,}000 \times 16 \times 0.0000061 \times 80$

$= 226{,}430$ lb. tension due to temperature change

Then: final total load =

226,430 lb. tension − 192,000 lb. compression = 34,430 lb. tension

Problems

Use the Appendix for necessary moduli of elasticity and coefficients of expansion.

12. A soft steel billet 2½ in. × 2½ in. is 30 ft. long. If the temperature changes from 1300°F. to 60°F, what is the total change in length?

13. A piece of gray cast iron is 2 in. long and 1 in. in diameter. What total load is required to decrease its length to 1.995 in.?

14. A hickory post is 3 ft. long and 4 in. × 4 in. in cross section. How much is the post shortened by a load of 16,000 lb.? What is the unit stress in the material? Use $E = 1{,}200{,}000$ lb. per sq. in.

15. A piece of lead pipe is stretched by a pull of 1000 lb. The pipe is 10 in. long and has an external diameter of 1.90 in. and an internal diameter of 1.50 in. How much does the piece elongate?

16. A wrought iron tie rod 15 ft. long and 1 sq. in. cross section area is placed in position, with the ends held by nuts as shown in Fig. 13, at a temperature of 60°F. The rod sustains a tensile pull of 18,000 lb. When the temperature rises to 200°F., what is the clearance between the nuts and the walls if the rod can expand? Is there any tension in the rod at the higher temperature?

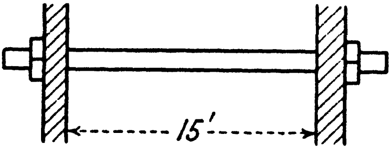


FIG. 13.

CHAPTER II

SHEAR

When forces are applied parallel to the cross section of a piece of material, one section of the piece tends to slide with respect to the adjacent section, thus causing the material to shear. One of the simplest examples of shearing action is the case of punching a hole in a steel plate.

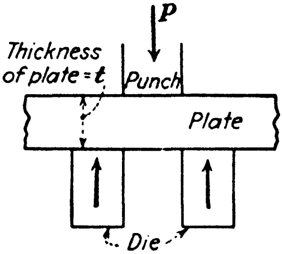


FIG. 14.

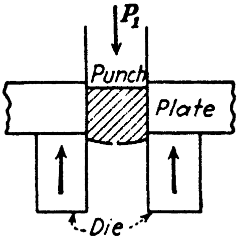


FIG. 15.

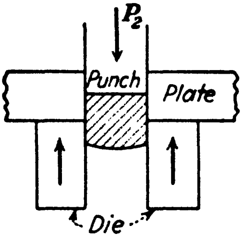


FIG. 16.

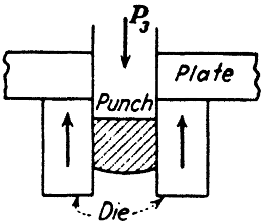


FIG. 17.

Figures 14, 15, 16, and 17 progressively show the action and movement which takes place. A force P is applied to the punch and the die pushes upward on the plate with an equal amount of force. In Fig. 15 the shearing action has started and the force P_1 required will be lessened as there is less metal remaining to be sheared. Figure 16, with a still smaller force P_2 required shows the punch about halfway through the plate. In Fig. 17, the shearing action is complete, so P_3 will be a very small amount, enough however, to overcome the friction between the punch and the plate.

Figure 18 shows the piece of the plate (slug) which has been removed. The shaded area is the *area* which has *sheared* and is called the *sheared area.* The student is cautioned against the use of the top circular area of the slug, since no failure, except a compression deformation, has taken place on this face. Hence, the *area* which has failed is the circumferential shaded area equal to the circumference of the slug times the thickness of the plate, or $A_s = \pi \times d \times t$ in which A_s represents the sheared area. (The sub s denotes that the failure was a shear failure.)

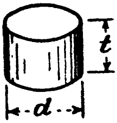


FIG. 18.

As an example: If the ultimate shearing strength of a steel plate is 42,000 lb. per sq. in., what force is necessary to punch a $\frac{3}{4}$-in.-diameter hole in a $\frac{5}{8}$ in. plate?

Now, the area which fails is the circumferential area of the disc to be removed, hence,

$$A_s = \pi dt = \pi \times \tfrac{3}{4} \times \tfrac{5}{8} = \tfrac{15}{32}\pi \text{ sq. in.}$$

then

$$P_s = S_s A_s = 42{,}000 \times \tfrac{15}{32}\pi = 61{,}800 \text{ lb. required to punch the hole}$$

Another example of shearing action occurs when the threads are stripped from a bolt. In Fig. 19 a nut has partially stripped the threads from the bolt and careful observation of the smooth shank will show that the *circumferential root area* has failed, leaving the shank threadless. Hence the sheared area will equal the circumference of the root (net) circle times the height of the nut, or

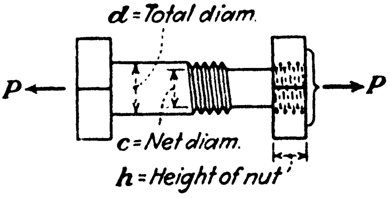


FIG. 19.

$$A_s = \pi \times c \times h$$

Example.—What force will be required to strip the threads on a $\frac{3}{4}$-in.-diameter bolt when the ultimate shearing strength of the material is 42,000 lb. per sq. in.?

From the Appendix, a $\frac{3}{4}$-in.-diameter bolt has a net (root) diameter of 0.620 in. and the height of the nut (from Table of Bolt Heads and Nuts) is $\frac{3}{4}$ in.

$$P_s = A_s S_s = \pi ch \times S_s = \pi \times 0.620 \times \tfrac{3}{4} \times 42{,}000$$

Then

$$P_s = 61{,}300 \text{ lb. required to strip the threads}$$

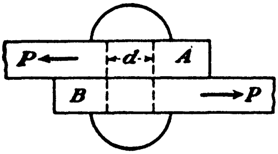


FIG. 20.

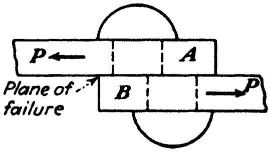


FIG. 21.

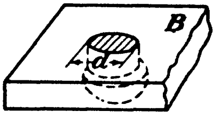


FIG. 22.

Figures 20 and 21 show two steel plates held together by a single rivet of diameter d. A force P is transmitted from plate A to plate B by the rivet. In Fig. 21 the rivet has been partially sheared due to the applied force P. After the rivet has been completely sheared, plate B is shown in Fig. 22, and it should be noticed that the *area* which has *failed* is the *cross section area* of the rivet. Hence,

$$A_s = \frac{\pi d^2}{4}$$

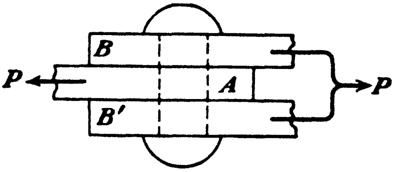


FIG. 23.

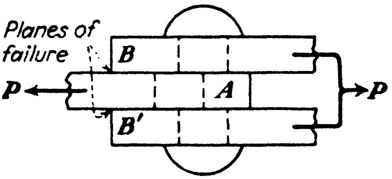


FIG. 24.

Since only one cross section fails, the failure is called *single* shear.

When two steel plates B and B', are riveted to a single plate A, the force P is transmitted from plate A to plates B and B' (Figs. 23 and 24). If the force becomes sufficiently large, the rivet will be sheared simultaneously on *two* different *cross sections:* between plates A and B, and between plates A and B'. This failure is called *double shear*. Hence,

$$A_s = 2 \times \frac{\pi d^2}{4}$$

These considerations which we have given to rivets hold equally true when bolts are used in place of rivets.

As an example of the shearing action in rivets, determine what force is required to cause a shear failure of a $\frac{3}{4}$-in. diameter rivet as shown in Fig. 20. The ultimate shearing stress of the material of which the rivet is made is 42,000 lb. per sq. in.

$$P_s = A_s S_s = \frac{\pi(\frac{3}{4})^2}{4} \times 42{,}000 = 0.4418 \times 42{,}000 = 18{,}560 \text{ lb. required}$$

Note: Use the Appendix (Areas of Square and Round Bars) for obtaining the cross section area of a rivet.

What force will shear a $\frac{1}{2}$-in.-diameter rivet as shown in Fig. 23 when the ultimate shearing strength of the rivet material is 42,000 lb. per sq. in.?

$$P_s = A_s S_s = 2\frac{\pi d^2}{4} \times S_s = 2\frac{\pi(\frac{1}{2})^2}{4} \times 42{,}000$$
$$= 2 \times 0.1963 \times 42{,}000 = 16.490 \text{ lb. required}$$

It should be understood by the student that the illustrative examples solved in the text are used on the basis of complete failure. In engineering practice, a structure, whether it be a bridge, machine, etc., is designed to be safe against failure. Hence the allowable unit stresses used are considerably lower than the ultimate unit stresses which exist when failure occurs. In other words, it is better to be **safe** than **sorry.** This applies to the design of machines as well as to any kind of work.

Table 4 contains a partial list of ultimate unit shearing stresses, and allowable unit shearing stresses used in practice. All figures represent *pounds per square inch.*

TABLE 4

Kind of material and use	Ultimate shearing strength	Allowable shearing strength
Steel shop rivet	40,000 to 47,000	13,500
Steel field rivet	40,000 to 47,000	10,000
Steel bolt	40,000 to 48,000	10,000
Structural steel shapes	42,000 to 48,000	12,000

Problems

17. An aluminum plate $\frac{3}{8}$ in. thick is to have a $\frac{13}{16}$-in. diameter hole punched in it. If 23,600 lb. is required to punch the hole, what is the ultimate unit shearing stress of the material?

18. Two plates as specified in problem 17 are to be riveted together with a $\frac{3}{4}$-in. diameter aluminum rivet. What safe load could the rivet transmit when the allowable unit shearing strength of the aluminum is 8000 lb. per sq. in.?

19. A ½-in. diameter steel bolt with a nut is tightened until the tensile force is 7500 lb. Will the bolt fail either in tension or by stripping the threads if the ultimate tensile strength of steel is 60,000 lb. per sq. in. and the ultimate shearing strength is 45,000 lb. per sq. in.?

20. Two plates are held together by two rivets. A force of 12,000 lb. is transmitted by the rivets from one plate to the other. What must be the diameter of the rivets if they are made of soft steel and if each has a factor of safety of 5? See Table 1 for the ultimate strength of soft steel.

21. Two steel plates are to be bolted to a single plate by one bolt ⅞ in. in diameter. What safe load can be transmitted if the allowable shearing strength of the bolt is 10,000 lb. per sq. in.?

22. If three bolts are used in problem 21, what is the safe load?

23. If six rivets are used in problem 18, what is the safe load?

CHAPTER III

RIVETED JOINTS

The primary object of a riveted joint is to transmit force from one steel plate to another, the plates being fastened together by rivets. If the plates are lapped as shown in Fig. 25 the joint is called a lap joint.

If the plates are butted together and one cover plate used, the joint is called a butt joint with one cover plate (Fig. 26).

When the plates are butted together and two cover plates applied, Fig. 27, the joint is called a butt joint with two cover plates.

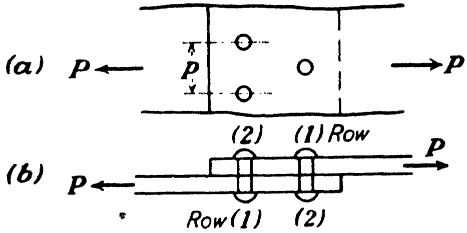


FIG. 25.

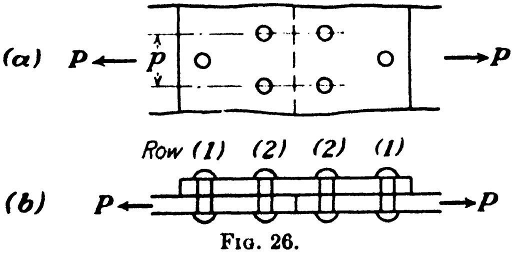


FIG. 26.

The designation of riveted joints also depends upon the *number* of *rows* of rivets in a lap joint or the number of rows on the right or left side of a butt joint. Thus Fig. 25 is properly designated a double riveted lap joint, Fig. 26 is a double riveted butt joint with one cover plate, and Fig. 27 is a triple riveted butt joint with two cover plates.

A row of rivets is a line of rivets in a direction perpendicular to the applied force *P*.

The **pitch** of the rivets is defined as the distance from the center of one rivet to the center of the next rivet in the same row. The rivets in any row are always spaced equally. When dealing with boiler or pipe joints where the joint is necessarily long, it is customary to use the *longest* distance between the centers of two adjacent rivets in the same row as the pitch, considering such a length of joint as a repeating section. Any number of repeating sections of equal strength may be placed end to end to form the complete joint. Thus, in Fig. 28, the repeating section *p*, of which three are shown, would be used as the basis of the design.

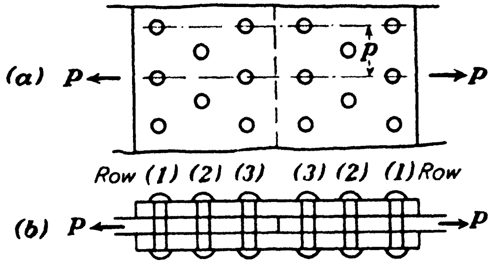


FIG. 27.

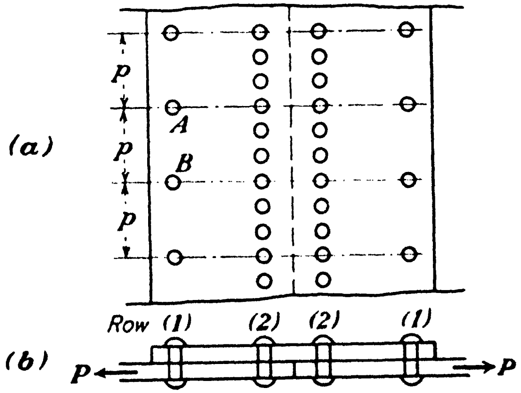


FIG. 28.

Where the joint is short, or where the total width of the plate is given, it is generally best to consider the entire plate as a unit.

Referring to Fig. 25, there are two whole rivets in the pitch distance; in Fig. 26, there are two whole rivets in the pitch distance; in Fig. 27, there are three whole rivets in the pitch distance; in Fig. 28 there are four whole rivets in the pitch distance.

It is advisable to deal with whole rivets in any pitch distance, although some of the whole rivets are composed of two halves of different rivets in the same row. In Fig. 28 the four whole rivets as stated consist of ½ of rivet *A* and ½ of rivet *B*, both of which are in row one, and two whole rivets and two half rivets in row two.

Riveted Joint Failures.—Any riveted joint may fail in one of the following manners:

1. By shearing the rivets (shear)
2. By tearing the plate between the rivet holes (tension)
3. By crushing the rivets, or the plate in front of the rivets which may be likened to elongating the rivet holes (bearing)

Shear.—As the question of rivet shear has been previously discussed, it will suffice to review shear as applied to riveted joints. By referring to Fig. 25, it will be seen that the number of rivets cut off in shear in the pitch distance is two, and each rivet is cut in single shear. Then,

$$P_s = 2\frac{\pi d^2}{4}S_s$$

Also in Fig. 26, two rivets in the pitch distance will be cut off in single shear and

$$P_s = 2\frac{\pi d^2}{4}S_s$$

In Fig. 27, there are three rivets in the pitch distance but as each rivet is in double shear,

$$P_s = 6\frac{\pi d^2}{4}S_s$$

In Fig. 28, there are four rivets in single shear in the pitch distance, and

$$P_s = 4\frac{\pi d^2}{4}S_s$$

If we let *n* represent the number of rivet areas in the pitch distance, the general equation of shear is

$$P_s = n\frac{\pi d^2}{4}S_s$$

Example.—Referring to Fig. 29, if the safe shearing stress is 15,000 lb. per sq. in., what safe load can the riveted joint support in shear?

As the plates are 8 in. wide, the total width (length of joint) shall be considered as a unit. There are four rivets in single shear on either side of the break in the main plate, hence,

$$P_s = n \frac{\pi d^2}{4} S_s$$

$$= 4 \frac{\pi(\frac{3}{4})^2}{4} 15{,}000$$

$$= 26{,}500 \text{ lb. safe load in shear}$$

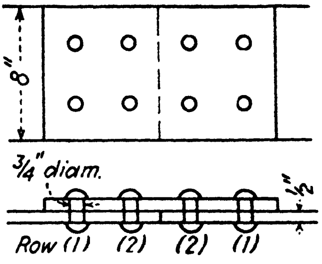


Fig. 29.

Tension.—Tearing the plate between the rivet holes constitutes a tension failure in a riveted joint. Using a single riveted lap joint as an example, where d = diameter of rivets, t = thickness of plates, and p = pitch distance. Figure 30*b* shows the joint of Fig. 30*a* cut on line A-A. The net *distance* between rivet holes is $(p - d)$ and the net *area* between the same holes is $(p - d)t$. It follows that

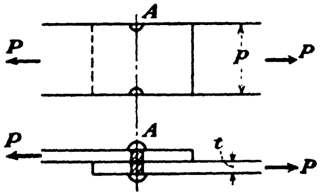


Fig. 30*a*.

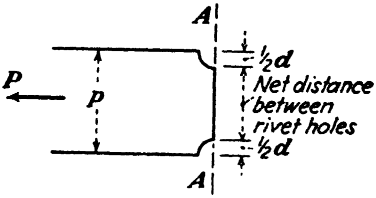


Fig. 30*b*.

$$P = AS$$

and

$$P_t = (p - d)tS_t$$

Example.—Let it be required to determine the safe load of the joint shown in Fig. 29 in tension in row 1 when the safe tensile stress is 18,000 lb. per sq. in. As the total width, 8 in., is considered the pitch distance, then

$$p = 8 \text{ in.}; \qquad d = \tfrac{3}{4} \text{ in.}; \qquad t = \tfrac{1}{2} \text{ in.};$$
$$S_t = 18{,}000 \text{ lb. per sq. in.}$$

$$P_t = (p - 2d)tS_t \text{ as two rivet holes occur in row 1}$$
$$P_t = (8 - 2[\tfrac{3}{4}])\tfrac{1}{2}18{,}000$$
$$= (6\tfrac{1}{2})\tfrac{1}{2}(18{,}000)$$
$$= 58{,}500 \text{ lb. safe load in tension}$$

Bearing.—The failure due to bearing is one where the rivet or the plate back of the rivet is crushed. If we consider a square pin (rivet) inserted in a plate of thickness t (Fig. 31) the area against which the pin pushes is equal to the thickness of the plate times the width d of the pin or dt. A similar condition prevails if the pin has a circular area, where it is customary to take a section through the center of the rivet perpendicular to the direction of the applied force. Then, this section has a rectangular area of height t and width d or the area is dt as above.

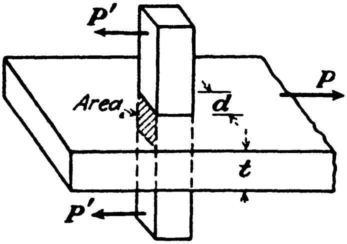


FIG. 31.

It should be noted that if the joint or the pitch distance has more than one rivet, all of the rivets will be crushed or all of the rivet holes will be elongated.

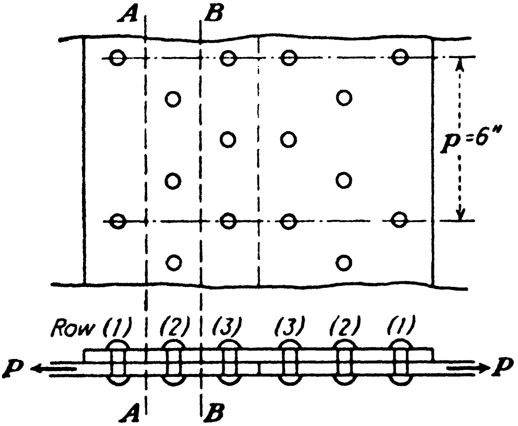


FIG. 32.

Example.—What is the safe load of the joint shown in Fig. 29 in bearing when the safe unit bearing stress is 20,000 lb. per sq. in.?

The general formula for bearing stress will be

$$P_b = ndtS_b$$

and as there are four rivet holes to be elongated,

$$\begin{aligned} P_b &= 4(\tfrac{3}{4})(\tfrac{1}{2})(20{,}000) \\ &= 30{,}000 \text{ lb. safe bearing load.} \end{aligned}$$

Example.—Figure 32 shows a typical boiler joint where the length of the repeating section is 6 in. The rivets are $\frac{3}{4}$ in. in diameter and the plates are $\frac{3}{4}$ in. thick. The safe stresses are:

$$\begin{aligned} S_s &= 8800 \text{ lb. per sq. in.} \\ S_t &= 11{,}000 \text{ lb. per sq. in.} \\ S_b &= 19{,}000 \text{ lb. per sq. in.} \end{aligned}$$

What maximum safe load can be applied to the joint (repeating section)?

Shear:

$$P_s = n\frac{\pi d^2}{4}S_s$$

There are five rivets in single shear.

$$\begin{aligned} P_s &= 5\frac{\pi(\frac{3}{4})^2}{4}8800 \\ &= 19{,}400 \text{ lb. safe shear load} \end{aligned}$$

Tension: considering tensile failure in row 1,

$$\begin{aligned} P_t &= (p - d)tS_t \\ &= (6 - \tfrac{3}{4})(\tfrac{3}{4})(11{,}000) \\ &= 43{,}300 \text{ lb. safe tensile load on row 1} \end{aligned}$$

Bearing:

$$P_b = n\,dtS_b$$

There are five rivet holes to be elongated.

$$\begin{aligned} P_b &= 5(\tfrac{3}{4})(\tfrac{3}{4})(19{,}000) \\ &= 53{,}500 \text{ lb. safe bearing load} \end{aligned}$$

It is now necessary to investigate the possible tensile failure in rows other than row 1, when fewer rivets are used in row 1 than in row 2 or 3.

From the results of a number of tests on full size riveted joints, it has been found safe to assume that each shear area transfers its proportionate part of the total load from the one main plate, to the other main plate in the case of lap joints or to the cover plates in the case of butt joints.

Referring to Fig. 32, it will be noted that all of the load *P* (per pitch) is applied to or reaches the main plate between the rivets in row 1. Since there are five shearing areas to transmit the load *P* to the cover plate, then

$$P_t = \tfrac{5}{5}(p - d)tS_t = 43{,}300 \text{ lb., as above on row 1}$$

Considering the line *A-A* in Fig. 33, since *one* rivet area lies between it and the left side of the joint, *one-fifth* of the total load *P* has been transferred by this rivet area to the cover plate. Hence $\tfrac{4}{5}P$ is left in the main plate and reaches the main plate on the line of rivets in row 2. In this row, there is

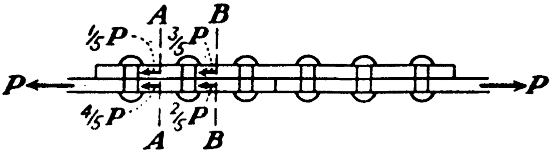


FIG. 33.

less plate area due to the two holes passing through the main plate, hence,

$$\tfrac{4}{5}P = (p - 2d)tS_t$$
$$\tfrac{4}{5}P = (6 - 2 \times \tfrac{3}{4})\tfrac{3}{4} \times 11{,}000$$
$$P_t = \tfrac{5}{4}(4\tfrac{1}{2})\tfrac{3}{4} \times 11{,}000$$
$$P_t = 46{,}400 \text{ lb. safe tensile load on row 2}$$

Considering the line *B-B* in Fig. 33, since *three* rivet areas lie between it and the left side of the joint, *three-fifths* of the total load *P* has been transferred by these rivet areas to the cover plate. Hence $\tfrac{2}{5}P$ is left in the main plate and reaches the main plate on the line of rivets in row 3. In this row, there is also less plate area than in row 1 due to the two holes passing through the main plate in row 3, hence,

$$\tfrac{2}{5}P = (p - 2d)tS_t$$
$$\tfrac{2}{5}P = (6 - 2 \times \tfrac{3}{4})\tfrac{3}{4} \times 11{,}000$$
$$P_t = \tfrac{5}{2}(4\tfrac{1}{2})\tfrac{3}{4} \times 11{,}000$$
$$P_t = 92{,}700 \text{ lb. safe tensile load on row 3}$$

Analyzing the tensile results for each of rows 1, 2, and 3, it will be seen that

for row 1

P_t = 43,300 lb. when the unit tensile stress is 11,000 lb. per sq. in.

for row 2

P_t = 46,400 lb. when the unit tensile stress is 11,000 lb. per sq. in., or a load P = 46,400 lb. could be used on the joint without producing a unit tensile stress greater than the allowable stress in the main plate on the second line of rivets.

for row 3

P_t = 92,700 lb. could be used on the joint without producing a unit tensile stress greater than the allowable stress in the main plate on the third line of rivets.

Summarizing, we have that the joint can carry 19,400 lb. without exceeding the safe shearing stress in the rivets. While greater loads could be applied to the joint and not exceed the safe bearing or tensile stresses, such as P_b = 53,500 lb., P_t = 43,300 lb. (row 1), P_t = 46,400 lb. (row 2), and P_t = 92,700 lb. (row 3), each one would cause a shearing stress greater than 8800 lb. and would, therefore, not be a safe load for all the stresses. The safe load for the joint is the *least load that it can carry safely*, or in this case 19,400 lb.

It should be noticed that when there are the same number of rivets in each row, the safe load can be calculated for the row in which the main plate transmits the entire load P. Thus for Fig. 29, rows 1 and 2 each contain two rivets, and as the plate in row 1 transmits the entire load P, then P_t for row 1 will be as given (58,500 lb.) and P_t for row 2 need not be calculated.

As a further illustration, let us assume that in Fig. 32, there is one rivet in each of rows 1 and 2 and two rivets in row 3, making *four* rivet areas, then

for row 1

$$\tfrac{4}{4}P = (p - d)tS_t$$
$$P = \tfrac{4}{4}(6 - \tfrac{3}{4})\tfrac{3}{4} \times 11{,}000$$
$$P_t = 43{,}300 \text{ lb.}$$

for row 2

$\tfrac{3}{4}P = (p - d)tS_t$ which need not be calculated as P is numerically greater than P_t for row 1.

for row 3

$$\begin{aligned} \tfrac{2}{4}P &= (p - 2d)tS_t \\ P_t &= \tfrac{4}{2}(6 - 2 \times \tfrac{3}{4})\tfrac{3}{4} \times 11{,}000 \\ P_t &= 74{,}200 \text{ lb.} \end{aligned}$$

P_b and P_s would be affected, since there are only four rivets but they have not been calculated as this illustration is used to emphasize the method of obtaining the safe tensile loads for each row.

Efficiency of Riveted Joints.—The efficiency of a riveted joint is defined as the ratio of the strength of the joint to the strength of the unpunched steel plate. If we apply a pull to a piece of steel plate with the same unit width (generally the pitch distance) throughout its length the plate can fail only in tension since there are no rivets to be sheared or crushed. The strength (total safe load) of a piece of steel plate then becomes

$$P = ptS_t$$

Then, the efficiency is

$$\text{Efficiency} = \frac{\text{safe load}}{\text{strength of plate}} \times 100\%$$

Referring to the example with Fig. 32, the safe load was determined as 19,400 lb. The strength of the steel plate is

$$\begin{aligned} P &= 6(\tfrac{3}{4})11{,}000 \\ &= 49{,}500 \text{ lb.} \end{aligned}$$

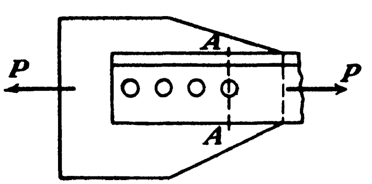
$$\text{Efficiency} = \frac{19{,}400}{49{,}500} \times 100 = 39.2\%$$

The ideal riveted joint is one in which the three total loads are equal or nearly so, since the efficiencies of the joint when using P_s, P_t, or P_b will be the same.

Structural Riveting.—The same general equations apply to structural riveting with certain modifications. The pitch distance with respect to a distance perpendicular to the line of application of the load is not necessary as all the rivets in the connection are counted.

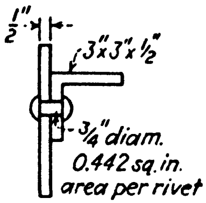
Example.—A 3-in. × 3-in. × $\frac{1}{2}$-in. angle is riveted to a steel plate (Fig. 34) by four $\frac{3}{4}$-in. rivets. The plate is $\frac{1}{2}$ in. thick. When $S_s = 15{,}000$ lb. per sq. in., $S_t = 20{,}000$ lb. per sq. in., and

$S_b = 32{,}000$ lb. per sq. in., what safe load (P) may be applied to the angle?



(a)

FIG. 34a.



(b)

FIG. 34b.

Shear: There are four rivets in single shear, hence

$$
\begin{aligned}
P_s &= 4\frac{\pi d^2}{4} S_s = 4(0.442)15{,}000 \\
&= 26{,}520 \text{ lb. safe load in shear}
\end{aligned}
$$

Bearing: There are four rivets in bearing, then

$$
\begin{aligned}
P_b &= 4\, dtS_b = 4(\tfrac{3}{4})\tfrac{1}{2}(32{,}000) \\
&= 48{,}000 \text{ lb. safe load in bearing}
\end{aligned}
$$

Tension: from the Appendix, the area of a 3-in. × 3-in. × $\frac{1}{2}$-in. angle section is 2.75 sq. in. The net area across any section (A-A) is the total area of the angle minus the rectangular cross section through the rivet hole (dt), or

$$
\begin{aligned}
2.75 - \tfrac{3}{4} \times \tfrac{1}{2} &= 2.75 - 0.38 = 2.37 \text{ sq. in.} \\
P_t &= A_t S_t \\
&= 2.37(20{,}000) \\
&= 47{,}400 \text{ lb. safe load in tension}
\end{aligned}
$$

The maximum safe load which can be applied to the angle is 26,520 lb. (shear).

In the following problems, these allowable stresses in pounds per square inch, which are the specifications of the American Institute of Steel Construction, shall be used for structural riveted joints:

$$
\begin{aligned}
S_s &= 15{,}000 \\
S_t &= 20{,}000 \\
S_b &= 32{,}000
\end{aligned}
$$

For other riveted joints, the following stresses in pounds per square inch shall be used:

$$S_s = 8800$$
$$S_t = 11{,}000$$
$$S_b = 19{,}000$$

Problems

24. Two steel plates, each 12 in. wide and ½ in. thick are riveted together with ¾-in. rivets in the form of a lap joint, with two rows of four rivets each. What safe load may be placed on the joint?

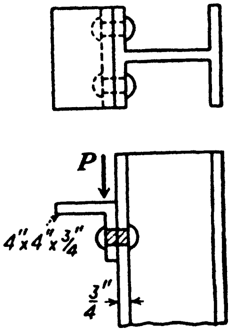


FIG. 35.

25. A steel angle 4 in. × 4 in. × ¾ in. is held by two ⅞-in. diameter rivets to a steel column (Fig. 35). Using the structural stresses, what load P may be placed on the angle? Consider shear and bearing only.

26. A butt joint with two cover plates has main plates of ⅝-in. steel, ¾-in. diameter rivets and the pitch equal to 3 in. There are three rows of rivets on each side of the joint and there is one rivet in each row in the pitch distance. What safe load will the repeating section stand? What is the efficiency of the joint?

27. If P_1 = 56,560 lb. in Fig. 36, which shows a structural connection, how many rivets are required to hold the 3-in. × 3½-in. × ⅜-in. angles to the gusset plate to be safe for shear? How many rivets are required to be safe in the bearing of the rivets against the gusset plate?

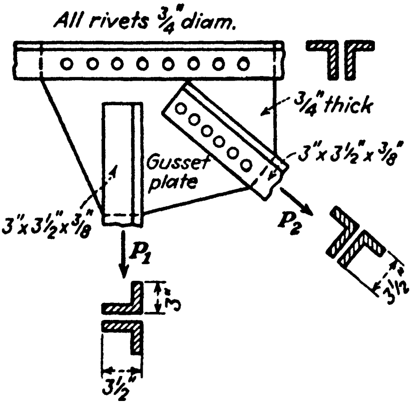


FIG. 36.

28. What unit tensile stress is developed in the angles when P_1 is 56,560 lb. (Fig. 36)?

29. What maximum value of P_2 may be used in Fig. 36 considering shear bearing, and tension? There are six rivets connecting the angles to the gusset plate.

30. In Fig. 37, the length of the repeating section is 9 in. What safe load will the joint stand in shear, in tension, and in bearing? Investigate for tension in rows 1 and 2.

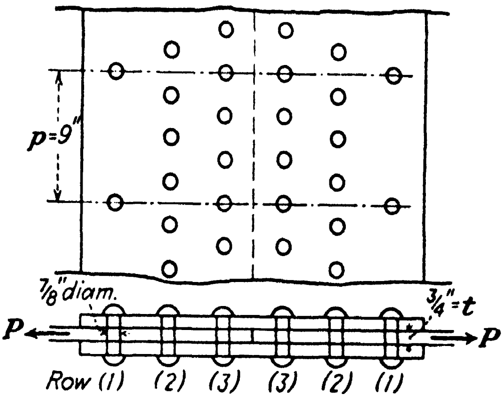


FIG. 37.

31. Referring to problem 30 and Fig. 37, what is the efficiency of the joint?

32. A triple riveted butt joint (one cover plate) with three rows of rivets each row of which contains one rivet in the repeating section has ¾-in.-diameter rivets, ⅝-in. plates and 3-in. pitch. What unit stresses in shear bearing, and tension will result if 15,000 lb. is applied to the joint?

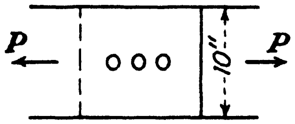


FIG. 38a.

33. Two steel plates 10 in. wide are to be riveted with three rivets to form a lap joint (Fig. 38a, b, c). The rivets are to be ⅞ in. in diameter, with the plates ¾ in. thick. If *shearing* stress only is considered, would you arrange the rivets as shown in Fig. 38a or as shown in Fig. 38c for maximum safe load?

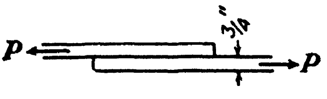


FIG. 38b.

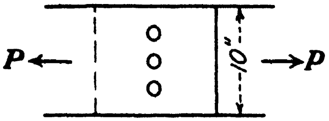


FIG. 38c.

34. Referring to problem 33 and considering *earing* only, would you arrange the rivets as shown in Fig. 38a or 38c for maximum safe load?

35. Referring to problem 33 and considering *tension* only, would you arrange the rivets as shown in Fig. 38a or 38c for maximum safe load?

CHAPTER IV

STRESSES IN THIN WALLED CYLINDERS—WELDS

The steam pressure which tends to rupture a boiler acts normally to the surface of the boiler as does the water pressure in water pipes. As a result, these pressures must not induce excessive stresses in the material from which the structures are made, or the elastic limit of the materials will be exceeded and failure will ultimately take place.

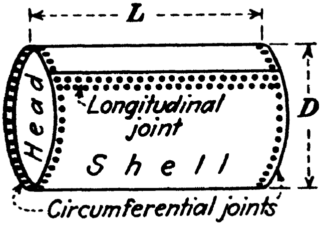


FIG. 39.

Boilers.—Figure 39 shows a boiler of diameter D and length L with the longitudinal and circumferential riveted joints. The longitudinal joint is used to fasten the shell and is parallel to the geometrical axis of the boiler. The circumferential joint is used to fasten the head of the boiler to the shell.

If we let R represent the steam pressure in pounds per square inch, then the total pressure acting on the head of the boiler is

$$P = AS$$

$$P = \frac{\pi D^2}{4} R$$

Rather than attempt to count the number of rivets in the circumferential joint, it is better to deal with the repeating section of a length equal to the pitch. To calculate the number of pitch distances around the circumference, it is necessary to divide the circumference by the pitch

$$\text{number of pitch distances} = \frac{\pi D}{p}$$

The pressure which comes to each pitch distance will be the total pressure on the head of the boiler divided by the number of pitch distances, or

$$\text{pressure per pitch} = \frac{P}{\text{number of pitch distances}}$$

$$= \frac{\dfrac{\pi D^2}{4} R}{\dfrac{\pi D}{p}} = \frac{RDp}{4}$$

Thus, the load (pressure) per pitch will be

$$P \text{ per pitch} = \frac{RDp}{4}$$

from which the stresses (S_s, S_t, or S_b) set up in the circumferential joint can be calculated.

Example.—A boiler 5 ft. in diameter works under a steam pressure of 200 lb. per sq. in. If the pitch of the rivets in the circumferential joint is 3 in., what is the amount of the load per pitch?

Solving the example step by step:

Total load P on the end of the boiler $= AS$

$$P = \frac{\pi D^2}{4} R = \frac{\pi (60)^2}{4} 200$$

$$= 565{,}480 \text{ lb.}$$

Number of pitch distances in the circumference

$$= \frac{\pi D}{p} = \frac{\pi 60}{3} = 62.8$$

$$\text{load per pitch} = \frac{P}{\text{number of pitch distances}}$$

$$= \frac{565{,}480}{62.8} = 9000 \text{ lb.}$$

To check by the formula:

$$P \text{ per pitch} = \frac{RDp}{4} = \frac{200 \times 60 \times 3}{4}$$

$$= 9000 \text{ lb.}$$

Example.—A closed cylinder, 3 ft. internal diameter, is made of $\frac{1}{4}$-in. steel plate. If the internal pressure is 300 lb. per sq. in., what is the tensile unit stress set up in the circumferential cross section of the shell (see Fig. 40*a*).

$$P = \frac{\pi D^2}{4} R$$

$$= \frac{\pi (36)^2}{4} 300 = 305{,}360 \text{ lb.}$$

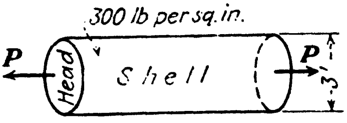


Fig. 40a.

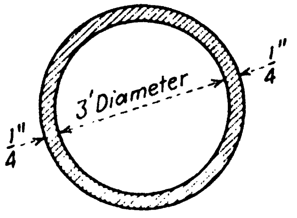


Fig. 40b.

Area of the ring of steel (Fig. 40b)

$$= \frac{\pi D^2}{4} - \frac{\pi D_1^2}{4}$$

$$= \frac{\pi (36.5)^2}{4} - \frac{\pi (36)^2}{4} = 28.47 \text{ sq. in.}$$

$$S_t = \frac{P}{A}$$

$$= \frac{305{,}360}{28.47} = 10{,}700 \text{ lb. per sq. in. tensile stress in the shell}$$

Figure 41 shows a half section of the shell of a thin cylinder, with a length L and a diameter D. The internal pressure (R) in pounds per square inch acts normal to the shell at every point. As the total pressure P' is the resultant pressure tending to break the shell at AB and CD, it is the sum of the components of all of the unit forces in a direction parallel to the resultant. The amount of the resultant pressure P' is obtained by multiplying the unit internal pressure by the projection of the area of the semi-circumference $ABECD$ on a plane, which is a rectangle with a length L and height D.

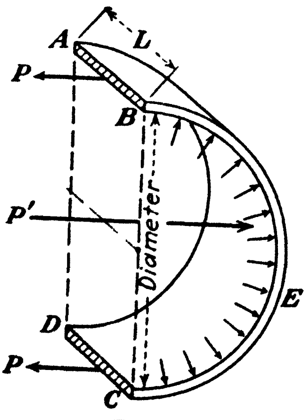


Fig. 41.

Therefore

$$P' = RDL$$

Now, the total pressure P' is resisted by two equal pressures P and P acting on areas AB and CD hence the force

$$P = \frac{P'}{2} = \frac{RDL}{2}$$

As the pitch distance only is the length which is considered, then

$$P = \frac{RDp}{2} = \text{load per pitch on the longitudinal riveted joint.}$$

Example.—A boiler, 5 ft. in diameter works under a steam pressure of 200 lb. per sq. in. If the pitch of the rivets in the longitudinal joint is 3 in., what is the total load per pitch?

Solving the example step by step:

Total load P' on the projected area of the semi-circumference $= AR$

$$\begin{aligned} P' &= RDp \\ &= 200 \times 60 \times 3 = 36{,}000 \text{ lb.} \end{aligned}$$

Total load P at the AB or CD area (Fig. 41) is $\frac{P'}{2}$

Therefore

$$P = \frac{P'}{2} = \frac{36{,}000}{2} = 18{,}000 \text{ lb. per pitch}$$

To check by the formula:

$$P = \frac{RDp}{2} = \frac{200 \times 60 \times 3}{2} = 18{,}000 \text{ lb. per pitch}$$

With reference to the preceeding example and the similar example where the load per pitch on the circumferential joint was solved to be 9000 lb., it is worth while to note that when the pitch distances on the longitudinal and circumferential joints are equal, the load per pitch on the circumferential joint is one-half the load per pitch on the longitudinal joint.

Example.—In Fig. 40, let it be required to determine the unit tensile stress developed in the longitudinal cross section of the shell. Considering the shell to be 1 ft. long (any unit length could be used), then

$$\begin{aligned} P &= \frac{RDL}{2} = \frac{300 \times 36 \times 12}{2} \\ &= 64{,}800 \text{ lb. per ft. of length} \end{aligned}$$

Area AB or CD (Fig. 41) $= 12 \times \frac{1}{4} = 3$ sq. in.

$$S_t = \frac{P}{A} = \frac{64{,}800}{3} = 21{,}600 \text{ lb. per sq. in. tensile stress}$$

on the longitudinal cross section of the shell.

Stand Pipes.—Figure 42 shows a vertical pipe filled with water. The water pressure in pounds per square inch at any depth is equal in all directions and is numerically equal to the weight of the vertical column of water above the depth considered.

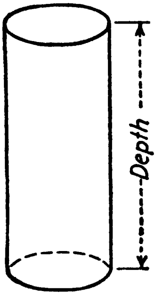


Fig. 42.

Water weighs 62.5 lb. per cu. ft. If the depth of water is 1 ft., the total weight of a column of water 1 ft. sq. and 1 ft. deep is

$$62.5 \times 1 \text{ cu. ft.} = 62.5 \text{ lb.}$$

Then the weight of water per foot of depth resting on 1 sq. in. is the weight resting on 1 sq. ft. divided by 144, or

$$\text{Weight} = \text{pressure} = \frac{62.5}{144} = 0.434 \text{ lb. per sq. in. per ft. of depth.}$$

Example.—What is the water pressure at the bottom of the standpipe when the depth of the water is 60 ft.?

$$\text{Pressure} = 0.434 \text{ lb. per sq. in. per ft. of depth}$$
$$\text{Pressure} = 0.434 \times 60 = 26.04 \text{ lb. per sq. in.}$$

This pressure, 26.04 lb. per sq. in., is corresponding to the steam pressure R in a boiler, except that the water pressure varies directly with the depth, while the pressure R is constant.

Example.—A steel water pipe is used to convey water under a head of 200 ft. The pipe is 1 ft. in diameter and $\frac{1}{4}$ in. thick. What is the pressure in the pipe and what is the unit tensile stress on the longitudinal cross section of the pipe.

$$\text{Pressure} = 0.434 \times 200 = 86.8 \text{ lb. per sq. in.} \ (= R)$$

Referring to Fig. 41, and using a length of 1 ft.:

$$P = \frac{RDL}{2} = \frac{86.8 \times 12 \times 12}{2}$$
$$= 6250 \text{ lb.}$$

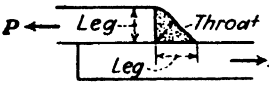
$$S_t = \frac{P}{A} \text{ where } A \text{ is the area } AB \text{ or } CD$$

$$S_t = \frac{6250}{12 \times \frac{1}{4}} = 2080 \text{ lb. per sq. in.}$$

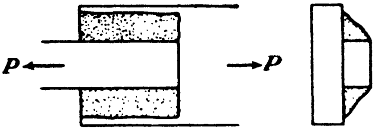
Welds.—Welding is the joining together by means of heat. Welding is generally done by the fusion method with an oxyacetylene torch or an electric arc. The welding material in the form of a slender rod melts and unites with the base metal to form the weld.

There are two main classes of welds:

1. Fillet welds shown in Figs. 43 and 44.
2. Butt welds shown in Figs. 45, 46 and 47.

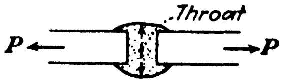


Transverse fillet weld
Fig. 43.

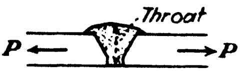


Side fillet weld
Fig. 44.

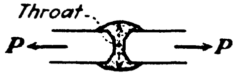
The plain butt weld is used when the plate thickness is $\frac{3}{8}$ in. or less; the single vee weld is used on plates $\frac{3}{8}$ in. and thicker; the double vee weld is used on thicker plates than the single vee



Plain butt weld
Fig. 45.



Single vee weld
Fig. 46.



Double vee weld
Fig. 47.

or where the work can be welded from both sides. The fillet welds and the plain butt welds require no machining before welding, and the fillet welds permit joining plates that are to be lapped.

The American Welding Society for fusion welding gives the following allowable unit stresses for the throat of the welds:

Shear	13,600 lb. per sq. in.
Tension	16,000 lb. per sq. in.
Compression	Same as base metal

The throat of the weld has been shown in Figs. 43 to 47 inclusive, and it is on this area that the strength of the weld depends. In a fillet weld, shown in Fig. 48, the lengths of the two legs are equal, making the cross section of the weld a 45° right triangle. If we assume a $\frac{1}{2}$-in. fillet weld where each leg is $\frac{1}{2}$ in., the throat dimension is

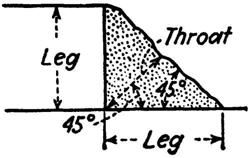


FIG. 48.

$$\frac{1}{2}\text{ in. } \cos 45° = 0.5 \times 0.707$$
$$= 0.3535$$

When the area of the throat per lineal inch of weld is

$$0.3535 \times 1 = 0.3535 \text{ sq. in.}$$

the shearing value of 1 lineal in. of $\frac{1}{2}$-in. fillet weld is

$$P = AS$$
$$P = 0.3535 \times 13{,}600 = 4800 \text{ lb.}$$

Similar values for fillet welds of different leg dimensions could be worked, but are tabulated as follows:

Leg of Fillet, Inches	Allowable Shearing Load, Pounds per Lineal Inch of Fillet Weld
$\frac{1}{8}$	1200
$\frac{1}{4}$	2400
$\frac{3}{8}$	3600
$\frac{1}{2}$	4800
$\frac{5}{8}$	6000
$\frac{3}{4}$	7200

Fillet welds placed *either* transverse *or* parallel to the direction of the load shall be considered under shear.

The essential factor in designing welds to resist or transmit forces is to have the center of gravity of the lengths of the welds coincide with the line of action of the forces.

Example.—Two $\frac{1}{2}$-in. plates 4 in. wide are to be welded with a double vee weld. What safe tensile force can be used, assuming no over run of welding material? The over run is the surplus welding material making the throat area greater than the cross section of the plate to be welded.

Allowable tensile stress through the throat of the weld

$$= 16{,}000 \text{ lb. per sq. in.}$$
$$\text{Area of plate} = 4 \times \tfrac{1}{2} = 2 \text{ sq. in.}$$
$$P_t = AS = 2 \times 16{,}000 = 32{,}000 \text{ lb. safe load.}$$

Example.—Two ¼-in. plates as shown in Fig. 49, are to be fillet welded with ¼-in. fillets. If $P = 31{,}200$ lb., design the welded connection. Allowable shearing load per lineal inch of ¼-in. fillet weld = 2400 lb. Let L = length of weld in inches.

$$L = \frac{P}{2400} = \frac{31{,}200}{2400} = 13 \text{ in.}$$

The design requires that 13 lineal inches of welding be used, which should be located so that its center of gravity coincides with the line of action of the force P. As the two sides A and B cannot provide more than 12 lineal inches, it is necessary to weld along line C which cares for 3 in., and divide the remaining length equally along A and B.

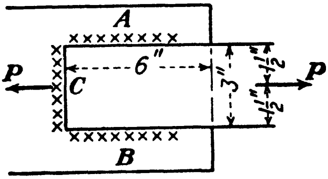


FIG. 49.

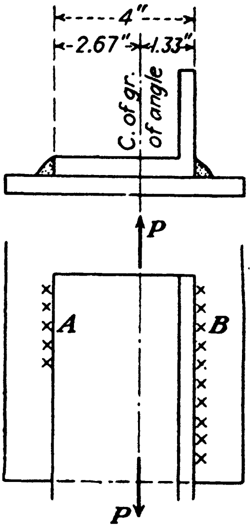


FIG. 50.

Then

$$A + B = 13 - C = 13 - 3 = 10 \text{ in.}$$
$$A = B$$

so

$$2A = 10 \text{ in. and } A = B = 5 \text{ in. long}$$

Example.—A 4-in. × 3-in. × ½-in. angle section is to have the 4-in. leg filet welded with ½-in. fillet welds to a steel plate. Using side welds only, find the length of weld necessary and give the lengths A and B in Fig. 50 when

$$P = 43{,}200 \text{ lb.}$$

From the Appendix, the centers of gravity of a 4-in. × 3-in. × ½-in. angle along the 4-in. leg is 1.33 in. from the corner of the angle. Allowable shear per inch of ½-in. fillet weld is 4800 lb. Length L of weld required:

$$L = \frac{P}{4800} = \frac{43{,}200}{4800} = 9 \text{ in.}$$

Now this 9 in. length must be divided into lengths A and B so that their center of gravity coincides with the line of the load P. This can be done most easily by moments about the load line, so that the moment of A equals the moment of B. Hence,

$$A(2.67) = B(1.33)$$

but

$$A + B = 9 \text{ in.} \qquad \text{and} \qquad A = 9 - B$$

then

$$\begin{aligned} (9 - B)(2.67) &= B(1.33) \\ 24.03 - 2.67B &= 1.33B \\ 4.00B &= 24.03 \\ B &= 6 \text{ in.} \\ A = 9 - B = 9 - 6 &= 3 \text{ in.} \end{aligned}$$

Example.—In the preceding problem, let it be assumed that length B cannot exceed 4 in. Then it is necessary to weld along side C which is 4 in. (Fig. 51), but in doing this the center of gravity of the A, B, and C welds must be kept on the line of the load P. Now

$$\begin{aligned} A + B + C &= 9 \text{ in.} \\ A + B + 4 &= 9 \text{ in.} \\ A + B &= 5 \text{ in.} \\ A &= 5 - B \end{aligned}$$

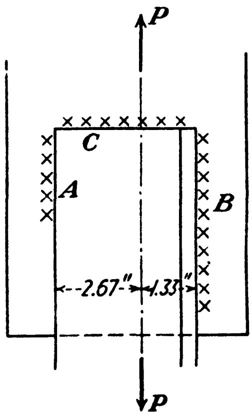


Fig. 51.

Again, taking moments about the line of the load remembering that the distance from the line of the load to the center of gravity of C is 2 in. − 1.33 in. = 0.67 in.

$$\begin{aligned} A(2.67) + C(0.67) &= B(1.33) \\ (5 - B)(2.67) + 4(0.67) &= B(1.33) \\ 13.35 - 2.67B + 2.68 &= 1.33B \\ 4.00B &= 16.03 \\ B &= 4 \text{ in.} \\ A = 5 - B = 5 - 4 &= 1 \text{ in.} \end{aligned}$$

Problems

36. What is the pressure per 6-in. pitch on the longitudinal joint of a 6-ft.-diameter boiler working under a steam pressure of 250 lb. per sq. in.?

37. In problem 36, what is the pressure per 3-in. pitch on the circumferential joint of the same boiler?

38. An 80-ft. stand pipe is filled with water. The pipe is 5 ft. in diameter and the plates are ½ in. thick. What is the maximum unit tensile stress per square inch on the longitudinal cross section of the plate?

39. The longitudinal boiler joint of a 5-ft.-diameter boiler is like the joint shown in Fig. 32. The steam pressure is 275 lb. per sq. in. Calculate the unit shearing stress and the unit bearing stress in the rivets. What is the unit tensile stress in the plate on row 1?

40. Figure 26 shows the repeating section of the circumferential joint on a 4-ft.-diameter boiler with 200 lb. per sq. in. steam. Pitch = 3 in.; diameter of rivets = ½ in.; t = ⅜ in. What are the values of S_s, S_t and S_b in the joint?

41. A horizontal water pipe 4 ft. in diameter is made from ¼-in. steel plate, with the longitudinal joint consisting of a single row of ½-in. diameter rivets at 2½-in. pitch. Considering that the rivets will fail in shear, where S_s = 12,000 lb. per sq. in., what maximum depth of water can be used in the pipe?

42. If the end of the pipe in problem 41 is closed by a valve, what unit tensile stress is placed on the circumferential cross section of the pipe?

43. A boiler 4½ ft. in diameter is made of ½-in. steel plate. The longitudinal joint is a double riveted butt joint with two cover plates, one rivet in each row in the pitch distance. Rivets are ¾ in. in diameter at 3-in. pitch. What maximum steam pressure can be used when S_s = 8800 lb. per sq. in.; S_t = 11,000 lb. per sq. in.; S_b = 19,000 lb. per sq. in.?

44. The circumferential joint of the boiler in problem 43 is to be a double riveted lap joint with ⅝-in. rivets at 3-in. pitch. Using the same allowable stresses, what maximum steam pressure can be used? There is one rivet in each row in the pitch distance.

45. A ⅜-in. steel plate 2 in. wide is to be lapped on a 4-in. wide steel plate and welded to it by ⅜-in. side fillet welds. What minimum length of weld on each side of the 2-in. plate can be used if the load applied as shown in Fig. 49 is 48,000 lb.?

46. If the 2-in. plate in problem 45 is to have a ⅜-in. transverse fillet weld in addition to the side welds, what minimum length of each side weld can be used?

47. A double vee weld is to be used to connect two 3-in. × ¾-in. steel plates. Assuming no over run of welding material, what total tensile load may be safely applied to the plates?

48. A 4-in. × 4-in. × ½-in. angle is to be side welded by ½-in. fillet welds to a steel plate. The load applied to the center of gravity of the angle section is 60,000 lb. What lengths of side welds will be necessary? Give a sketch of the locations and approximate lengths of the welds.

49. A 3½-in. × 4-in. × ⅝-in. angle is to have the 3½-in. leg welded by ⅝-in. transverse and side fillet welds to a plate. The load to be carried is 72,000 lb. applied to the center of gravity of the 3½-in. leg. Calculate the side lengths of the welds and give a sketch of the locations of the welds.

50. What safe load may be applied to the welded joint shown in Fig. 52?

51. Fig. 53 shows the cross section of a 15-in. diameter pipe made of $\frac{3}{8}$-in. plate. A single vee weld is used on the longitudinal seam. What steam pressure may be safely used in the pipe? Assume a unit length of pipe of one foot and that the weld will fail in tension under the action of the steam.

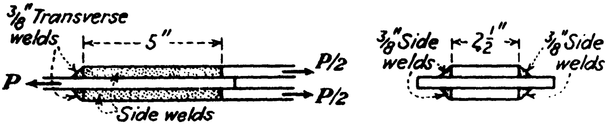


FIG. 52.

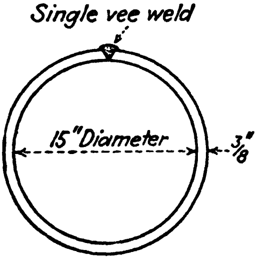


FIG. 53.

CHAPTER V

TORSION

Forces which cause a bar to twist about its central axis are called torsional forces. The wheel and axle, pulleys and gears are examples of machines which apply torsional forces to shafts. These forces casue an external twisting moment (**torque**) about the center of the shaft which is resisted by the moment of the internal shearing stresses.

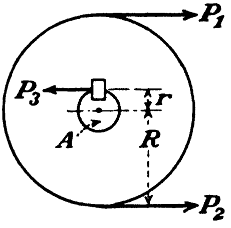


FIG. 54.

The pulley in Fig. 54 has a radius $= R$ and the belt pulls are P_1 and P_2 where P_1 is larger than P_2. The pulley is keyed to the shaft A, so that the resisting shear P_3 acts at the circumference of the shaft. The external **torque** is the sum of the moments of P_1 and P_2 about the center of the shaft =

$$P_1R - P_2R = (P_1 - P_2)R$$

The internal resisting moment is the moment of P_3 about the center of the shaft $= P_3r$

As the external torque equals the internal resisting moment.

$$(P_1 - P_2)R = P_3r$$

Example.—The belt pulls, $P_1 = 600$ lb. and $P_2 = 300$ lb., act at the circumference of a 20-in.-diameter pulley. The pulley is keyed to a 2-in.-diameter shaft by a key ¼ in. wide and 1 in. long. What is the *unit shearing* stress in the key?

$$\begin{aligned} \text{The external torque} &= (P_1 - P_2)R \\ &= (600 - 300)10 = 3000 \text{ in.-lb.} \\ \text{The internal moment} &= P_3r \\ &= P_3 \times 1 = 1P_3 \text{ in.-lb.} \end{aligned}$$

Equating the external torque and the internal moment

$$3000 \text{ in.-lb.} = 1P_3 \text{ in.-lb.}$$

from which

$$P_3 = 3000 \text{ lb.}$$

Hence, the force tending to shear the key is 3000 lb. The cross section of the key relative to the surface of the shaft is

$$\tfrac{1}{4} \text{ in.} \times 1 \text{ in.} = \tfrac{1}{4} \text{ sq. in.} \quad \text{(see Fig. 55)}$$

In Chap. II, it was shown that

$$P_s = S_s A_s$$

hence

$$P_s = P_3 = 3000 \text{ l b. and } A_s = \tfrac{1}{4} \text{ sq. in.}$$

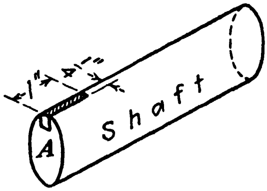


FIG. 55.

then

$$S_s = \frac{P_s}{A_s} = \frac{3000}{\frac{1}{4}} = 12{,}000 \text{ lb. per sq. in. (shearing stress in the key)}$$

When the ends of the two shafts are to be connected by a flange coupling, the twisting moment (torque) is transmitted from one shaft to the other. This twisting moment develops shearing stresses in the bolts. In Fig. 56 the side and cross section views of the coupling are shown, in Fig. 57, the end view is shown, and in Fig. 58 a pictorial view is shown. The radius of the circle on which the bolts are placed is r. If we let P_s (Fig. 52) equal the shearing strength of each bolt in pounds the total shearing strength of all of the bolts will be $P_s \times n$ where n is the number of bolts in the coupling. As

$$P_s = A_s S_s = \pi \frac{d^2}{4} S_s \text{ for one bolt}$$

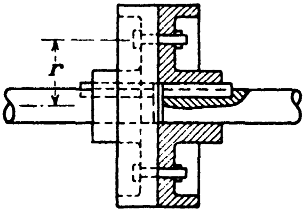


FIG. 56.

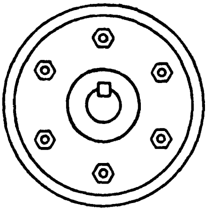


FIG. 57.

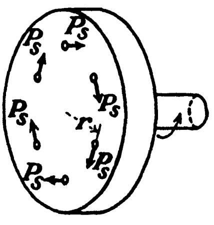


FIG. 58.

then total

$$P_s = n\frac{\pi d^2}{4}S_s$$

The resisting moment of each bolt is the moment of the shearing strength about the center of the shaft which is

$$P_s r$$

and the resisting moment of the shearing strength of all of the bolts is

$$nP_s r = n\frac{\pi d^2}{4}S_s r$$

The external torque T, which is to be transmitted by means of the bolts in the coupling, will be equal to the resisting moment, hence

$$T = n\frac{\pi d^2}{4}S_s r$$

Example.—A flange coupling is to transmit a torque of 8000 ft.-lb. The allowable shearing stress in the bolts is 10,000 lb. per sq. in., and the bolts are set in a circle with a 5-in. radius. How many $\frac{3}{4}$-in. bolts are required?

$$T = n\frac{\pi d^2}{4}S_s r$$

$$8000 \times 12 = n\frac{\pi(\frac{3}{4})^2}{4} \times 10{,}000 \times 5$$

Note: The torque must be changed to inch-pounds by multiplying by 12 in. per foot. Then

$96{,}000 = n(0.442) \times 10{,}000 \times 5$ (using the tables for the *total* area of a $\frac{3}{4}$-in. bolt)

from which

$$n = \frac{96{,}000}{0.442 \times 10{,}000 \times 5} = 4.35 \text{ bolts}$$

Therefore, five bolts are required, as it is obviously impossible to use a partial bolt.

It has been shown that

$$\text{hp.} = \frac{\text{work}}{33{,}000 \times t}$$

When a force of P is applied to the circumference of a pulley (Fig. 59) the amount of work done during *one revolution* of the pulley is

$$\text{work} = PS = P2\pi R$$

If the pulley is rotating at N revolutions per minute, the work done per minute will equal the work per revolution times the revolutions per minute which is power:

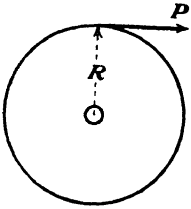


FIG. 59.

$$\text{work (per minute)} = P2\pi RN = \text{power}$$

But torque T is equivalent to the moment of the force P about the center of the shaft, hence

$$T = RP$$

then

$$\text{Power} = 2\pi RPN = 2\pi TN$$

and

$$\text{hp.} = \frac{2\pi NT}{33{,}000}$$

where hp. is the horsepower due to torque T when the pulley or shaft is rotating at N revolutions per minute.

Note: The use of T in this equation requires that the units of T must be *foot-pounds* to agree with the 33,000 *ft.-lb.* per min.

Example.—A flange coupling has six $\frac{3}{4}$-in. bolts set on a 10-in.-diameter circle. The allowable unit shearing stress in the bolts is 10,000 lb. per sq. in. If the shafts are rotating at 150 r.p.m., what horsepower can be transmitted by the coupling?

$$\begin{aligned} T &= n\frac{\pi d^2}{4} S_s r \\ &= 6 \times 0.442 \times 10{,}000 \times 5 \\ &= 132{,}600 \text{ in.-lb.} = \frac{132{,}600}{12} = 11{,}050 \text{ ft.-lb.} \end{aligned}$$

then

$$\begin{aligned} \text{hp.} &= \frac{2\pi NT}{33{,}000} \\ &= \frac{2\pi \times 150 \times 11{,}050}{33{,}000} = 316 \end{aligned}$$

Example.—A 4-in.-diameter shaft is keyed to a pulley. The key is 6 in. long and 0.3 in. wide. When the allowable unit shearing stress in the key is 12,500 lb. per sq. in. and the shaft is rotating at 200 r.p.m., what horse power can be transmitted?

The total shearing strength of the key is

$$\begin{aligned} P_s &= A_sS_s = 6\text{ in.} \times 0.3\text{ in.} \times 12{,}500\text{ lb. per sq. in.} \\ &= 22{,}500\text{ lb.} \end{aligned}$$

The resisting moment of the shearing strength equals the external torque:

$$\begin{aligned} T &= P_sr \\ &= 22{,}500 \times 2 = 45{,}000\text{ in.-lb.} \end{aligned}$$

and

$$T = \frac{45{,}000}{12} = 3750\text{ ft.-lb.}$$

then

$$\begin{aligned} \text{hp.} &= \frac{2\pi NT}{33{,}000} \\ &= \frac{2\pi \times 200 \times 3750}{33{,}000} = \frac{1{,}500{,}000\pi}{33{,}000} \\ &= 142.8 \text{ which can be transmitted safely} \end{aligned}$$

Problems

52. A pulley is keyed to a 3-in.-diameter shaft by a key 3 in. long and 0.28 in. wide. The allowable unit shearing stress of the steel key is 12,000 lb. per sq. in. How much torque will the key transmit safely?

53. A belted pulley (Fig. 60) is keyed to a 2-in.-diameter shaft by a key $\frac{3}{16}$ in. wide. How long must the key be to safely withstand an allowable unit shearing stress of 12,500 lb. per sq. in.?

54. How many horsepower is the shaft of problem 53 transmitting at 99 r.p.m.?

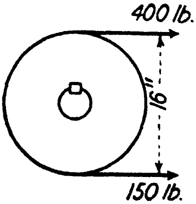


FIG. 60.

55. A flanged coupling is bolted together by four $\frac{1}{2}$-in. bolts arranged on the circumference of an 8-in.-diameter circle. The flanges are keyed to a 2-in. diameter shaft with keys 7 in. long by $\frac{3}{4}$ in. wide. The allowable unit shearing stress in the bolts is 10,000 lb. per sq. in. What is the total torque transmitted by the bolts?

56. In problem 55 what is the unit shearing stress in the keys?

57. In problem 55 how many horsepower are transmitted if the shafts are rotating at 198 r.p.m.?

58. A pulley 40 in. in diameter transmits 40 horsepower. It is keyed to a shaft and the shaft is coupled to another shaft by a flanged coupling with ½-in. bolts set in a 4-in. radius circle. The shafts rotate at 165 r.p.m. The lesser belt pull P_2 is 100 lb. What is the amount of the greater belt pull P_1?

59. In problem 58 how many bolts are required in the couplings when the allowable unit shearing stress in the steel bolts is 10,000 lb. per sq. in.?

60. If the bolts are replaced by a material having an allowable unit shearing stress of 8000 lb. per sq. in., how many bolts are necessary?

Shafts.—When forces are applied to shafts through the medium of pulleys, gears, etc., they tend to twist the shaft as shown in Fig. 61. The line AB was straight before the forces P and P, each acting at a lever arm of R, were applied. (The left end of the shaft is held rigid.) After the application of the forces, P and P, the line AB assumes the position indicated by the line AB'. Any cross section of the shaft tends to slip by or shear off with respect to the next adjacent cross section. Such a cross section is shown at the right end of Fig. 62.

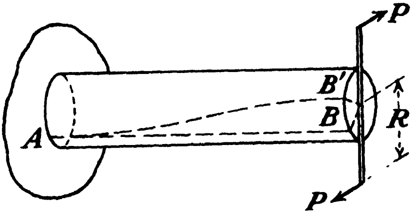


FIG. 61.

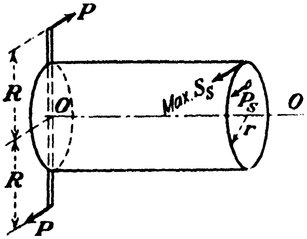


FIG. 62.

The external applied torque sets up within the shaft internal resisting forces, of which *one*, P_s, is shown in Fig. 62. Now the external torque which is PD or $2PR$ must be equal to the sum of the moments of the internal resisting shearing forces. By the use of mathematics, which permits us to add together the total resisting moments, their sum is found to equal SJ/r.

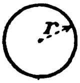
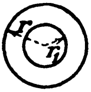
Equating the external torque and the sum of the internal resisting shears

$$T = \frac{SJ}{r} \tag{1}$$

If such a shaft is twisted until it fails, the shear break is a progressive fracture starting at the surface of the shaft and gradually, but very rapidly, shearing until the central axis O-O is reached. Thus, the *maximum tendency* to *shear* is at the *surface* of the shaft and the tendency at the center of the shaft is zero. In eq. (1)

T = external applied torque (inch-pounds)
S = maximum unit shearing stress at the outside surface of the shaft (pounds per square inch)
J = polar moment of inertia of the cross section of the shaft Values of J for various shaped cross sections are given in inches.[4] Table 5
r = outside radius in inches

TABLE 5

Type of shaft	Value of J
 Solid circular	$\frac{\pi r^4}{2}$
 Hollow circular	$\frac{\pi}{2}(r^4 - r_1{}^4)$

Example.—A solid circular steel shaft 2 in. in diameter is twisted by a force P acting 20 in. from the center of the shaft. The allowable unit shearing strength of the steel is 12,000 lb. per sq. in. What force P can be applied?

$$T = \frac{SJ}{r}$$

From Table 5

$$J = \frac{\pi r^4}{2}$$

hence

$$P \times 20 = \frac{12{,}000 \times \frac{\pi(1)^4}{2}}{1}$$

from which

$$P = \frac{12{,}000 \times \pi(1)^4}{20 \times 1 \times 2} = 942 \text{ lb.}$$

Example.—A hollow shaft is twisted by the link belt pulls, $P_1 = 5000$ lb. and $P_2 = 500$ lb. What is the maximum unit shearing stress in the hollow shaft (Fig. 63)?

For a hollow shaft:

$$J = \frac{\pi}{2}(r^4 - r_1^4) = \frac{\pi}{2}[(2)^4 - (1)^4]$$

$$J = \frac{\pi}{2}(16 - 1) = \frac{\pi 15}{2} = 23.56 \text{ in.}^4$$

$$T = \frac{SJ}{r}$$

$$T = (P_1 - P_2)R = (5000 - 500)20 = 90{,}000 \text{ in.-lb.}$$

then

$$90{,}000 = \frac{S \times 23.56}{2}$$

from which

$$S = \frac{90{,}000 \times 2}{23.56} = 7640 \text{ lb. per sq. in.}$$

In comparing the relative strengths of two shafts, the torque T is a measure of the strength of any shaft. Therefore, the comparison can be made between their torques, providing that the safe unit shearing stress has not been exceeded for either shaft.

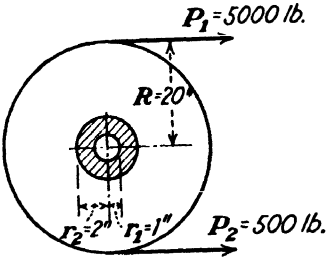


FIG. 63.

Example.—A hollow shaft has an external radius of 1 in. and an internal radius of 0.6 in. The strength of the shaft is to be compared with the strength of a solid shaft having the same cross section area. The allowable unit shearing stress for each shaft is 12,000 lb. per sq. in.

Area of hollow shaft $= \pi r^2 - \pi r_1^2 = \pi(1)^2 - \pi(0.6)^2 = 0.64\pi$ sq. in.
Area of hollow shaft = area of solid shaft

$$0.64\pi = \pi r^2$$

$$r^2 = 0.64$$

$$r = \sqrt{0.64} = 0.8 \text{ in.}$$

Hollow Shaft	Solid Shaft
$J = \frac{\pi}{2}(r^4 - r_1^4) = \frac{\pi}{2}[(1)^4 - (0.6)^4]$	$J = \frac{\pi r^4}{2} = \frac{(0.8)^4}{2}$
$= \frac{\pi}{2}(1 - 0.1296) = 1.367 \text{ in.}^4$	$= \frac{\pi}{2}(0.4096) = 0.644 \text{ in.}^4$
$T_1 = \frac{SJ}{r}$	$T_2 = \frac{SJ}{r}$
$T_1 = \frac{12{,}000 \times 1.367}{1}$	$T_2 = \frac{12{,}000 \times 0.644}{0.8}$
$T_1 = 16{,}404$ in.-lb.	$T_2 = 9660$ in.-lb.

Hence

$$\frac{T_1}{T_2} = \frac{16{,}404}{9660} = 1.698$$

Thus, the hollow shaft is 1.698 times as strong as a solid shaft, with an equal cross section area, when made of the same material.

Previously in this chapter, it was shown that

$$\text{hp.} = \frac{2\pi NT}{33{,}000}$$

where T = torque in foot-pounds. Therefore, if it is possible to determine the amount of torque which a shaft can transmit, and the speed of the shaft in r.p.m. is known, the horsepower of the shaft can be calculated.

Example.—In the problem immediately above, how many horsepower can the hollow shaft transmit when it is rotating at 150 r.p.m.?

$$T = \frac{16{,}404 \text{ in.-lb.}}{12 \text{ in. per ft.}} = 1{,}367 \text{ ft.-lb.}$$

Then

$$\text{hp.} = \frac{2\pi NT}{33{,}000} = \frac{2\pi \times 150 \times 1367}{33{,}000}$$

$$\text{hp.} = 39$$

At what speed must the solid shaft rotate to transmit the same horsepower as the hollow shaft?

$$T = \frac{9660 \text{ in.-lb.}}{12 \text{ in. per ft.}} = 805 \text{ ft.-lb}$$

$$\text{hp.} = \frac{2\pi NT}{33{,}000}$$

Then

$$39 = \frac{2\pi N \times 805}{33{,}000}$$

from which

$$N = \frac{39 \times 33{,}000}{2 \times 805} = 254.5 \text{ r.p.m.}$$

Problems

61. A 3-in. diameter solid steel shaft is used to transmit 24,000 in.-lb. of torque at 99 r.p.m. What is the maximum unit shearing stress in the shaft?

62. In problem 61, how many horsepower does the shaft transmit? Compare the horsepower of the following three solid shafts when making 120 r.p.m. The allowable unit shearing strength of the material is 10,000 lb. per sq. in.

63. When the diameter is 2 in., how many horsepower is transmitted?

64. When the diameter is 3 in., how many horsepower is transmitted?

65. When the diameter is 4 in., how many horsepower is transmitted?

66. An hydraulic turbine is directly connected to a generator by a solid shaft 25 in. in diameter. If it is rotating at 60 r.p.m., and the turbine has a maximum horsepower of 12,000, what is the maximum unit shearing stress in the shaft?

67. A hollow shaft, with outside and inside diameters 18 in. and 12 in., respectively, transmits 10,000 hp. at 99 r.p.m. What is the maximum unit shearing stress in the shaft?

CHAPTER VI

BEAMS, SHEAR AND MOMENT DIAGRAMS

Types of Beams.—Beams are usually classified by the number of supports. A beam which is fixed or supported at only one end is called a **cantilever beam.** Figure 64 is an example of a cantilever beam supporting a single concentrated load at the free end *B*.

When the beam is supported at each end, as shown in Fig. 66, the beam is called a **simple beam.** The simple beam is supporting only a single concentrated load.

Bending.—In addition to the simple stresses already studied, the stress due to bending must next be considered. Bending action is present in all beams and it brings about a condition of all three of the simple stresses of tension, compression and shear.

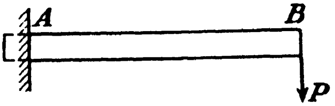


FIG. 64.

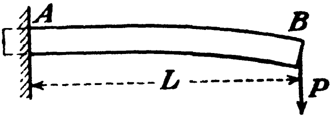


FIG. 65.

Figure 64 shows a cantilever beam with one end fixed and with the other end supporting a load *P*. This load *P* will now bend the beam somewhat as shown in Fig. 65. In order for the beam to bend this way the upper side must stretch and consequently be in tension, while the lower side must contract and be in compression. The amount of this tension and compression will, of course be in proportion to the amount of bending.

Axis of Rotation.—Since this beam in Figs. 64 and 65 is fixed at the point *A*, the force *P* is tending to make the beam rotate about the point *A*. If there were a hinge at *A* and the force *P* was large enough, there is no question but that the beam would swing about this point. In the same way if there were a hinge at any point in the beam the force *P* would cause rotation about this point. Thus we say that when a force is acting on a beam it will tend to cause rotation about various points in the beam. When the turning or rotating effect is being considered for any section of the beam, then this section is called the axis of rotation.

Figure 66 shows a simple beam supported at the ends, with a concentrated load P at the middle. The load P will cause the beam to bend (sag) as shown in Fig. 67. The bending causes the upper side of the beam to shorten while the lower side is lengthened. If the cross section of the beam is symmetrical (has the same shape above the line AB as below) about a horizontal plane half-way from the top to the bottom, as in the case of a rectangular, square, circular, or I-beam, there will be a surface AB in each beam where the beam is neither shortened nor lengthened.

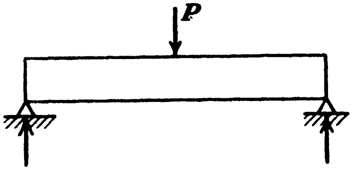


FIG. 66.

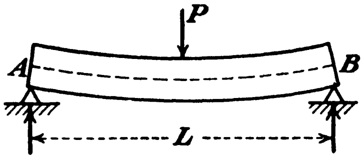


FIG. 67.

As a piece of material which is shortened is said to be in compression, and a piece which is lengthened is said to be in tension, the plane AB which is not deformed is called the **neutral axis,** as there is no stress on this plane due to the bending.

Shear in Beams.—In beginning the subject of beams, it was stated that a shearing stress was present in a beam as well as a tension and a compression. Figure 68 shows a beam with forces acting down and the reactions holding it up. It is apparent that there is a tendency for the beam to shear as shown by the dotted lines in the illustration.

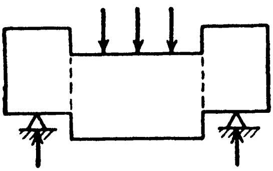


FIG. 68.

In most cases this shearing stress is so much smaller than the stress due to tension and compression that it can be ignored. However, if the beam is very short, then the shearing stress may be larger than the tension or the compression in which case it must be considered in designing the beam.

The amount of shear at any section will be the algebraic sum of all the forces acting on *either* side of that section. It is customary to calculate the shear from the forces acting on the *left* of the section. In calculating shear in a beam, those forces acting downward are called negative and those acting upward are called positive.

Let it be required to calculate the shear for different sections along the beam in Fig. 69. If we assume *any* section between the reaction R_1 and the force P_1, there will be only one force acting to the left of this section and this force will be the reaction R_1 of 527 lb. Hence, the shear for this section of the beam will be 527 lb.

For any section between the forces P_1 and P_2 there will be two forces acting to the left of P_2, R_1 acting up and, therefore, positive, and P_1 acting down, and, therefore, negative. Hence, the shear for any point in this section will be 527 − 500 or 27 lb.

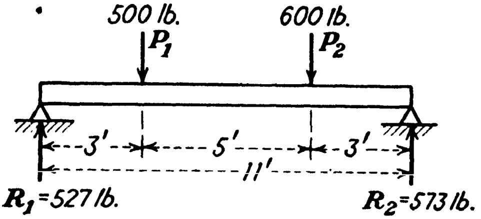


FIG. 69.

For a section of the beam between P_2 and R_2 there will be three forces, R_1 acting up and therefore positive, and P_1 and P_2 acting down and therefore negative. Hence, the shear for any section in this part will be

$$527 - 500 - 600 = -573 \text{ lb.}$$

The positive or negative sign in values for shear may be assumed to show that the external forces tend to slide the left portion upward or downward with respect to the right portion of the beam at the section under consideration.

For example, at any section from R_1 to P_2, the sum of the forces to the left of the section is positive and the portion of the beam to the left of the section tends to shear off and slide upward with respect to the portion of the beam to the right. From P_2 to R_1, the reverse is true.

Shear Diagram.—It is possible now to draw a shear diagram. The illustration of Fig. 69 has been recopied as Fig. 70 and a shear diagram drawn. In drawing the shear diagram, positive shear was marked off *above* the horizontal line and negative shear was laid off *below* the horizontal line.

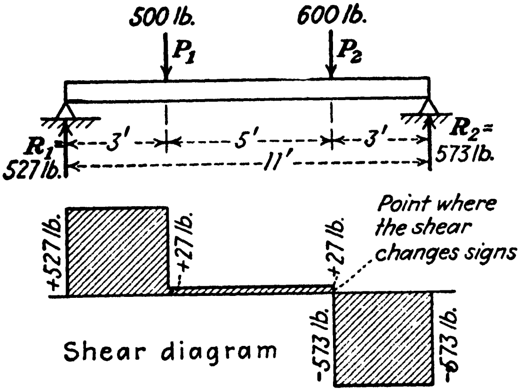


FIG. 70.

Example.—A beam 16 ft. long has a uniform load of 80 lb. per ft. and a load of 500 lb. 6 ft. from the left end. Draw the shear diagram and locate the point at which the shear changes sign (shear zero).

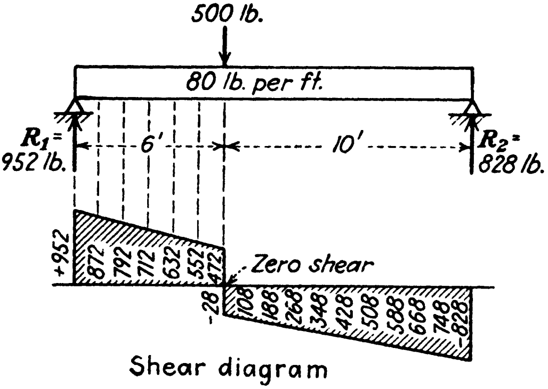


FIG. 71.

Solution.—First make a sketch of the beam as in Fig. 71, and then solve for the reactions R_1 and R_2.

$$R_1 \times 16 = (80 \times 16 \times 8) + (500 \times 10)$$

from which

$$R_1 = 952 \text{ lb.}$$

Likewise

$$R_2 \times 16 = (80 \times 16 \times 8) + (500 \times 6)$$

from which

$$R_2 = 828 \text{ lb.}$$

As a check on the values of R_1 and R_2:

$$\begin{aligned} R_1 + R_2 &\text{ must equal } 16 \times 80 + 500 \\ 952 + 828 &\text{ must equal } 16 \times 80 + 500 \\ 1780 &= 1780 \end{aligned}$$

∴ the values of R_1 and R_2 as determined above are correct.

The next step is to calculate the shear diagram. At R_1 we have the upward force of 952 lb. and no downward force so that the shear here is plus 952. At one foot to the right of R_1 the forces acting to the left will be R_1 acting up and one foot of the beam or 80 lb. acting down. Hence, the shear will be $952 - 80 = 872$ lb. Likewise, for a point 2 ft. from R_1 the shear will be

$$952 - (2 \times 80) = 792 \text{ lb.}$$

In this way the shear has been worked out and the diagram drawn in Fig. 71. The shear changes sign under the load of 500 lb. so this must be the point of zero shear.

The shear at any section of a cantilever beam is calculated in the same manner as in a simple beam. However, in order that the correct shear signs appear, it is necessary to consider that the cantilever beam always has the *supporting wall* and *reaction* on the *right* as shown in Fig. 72.

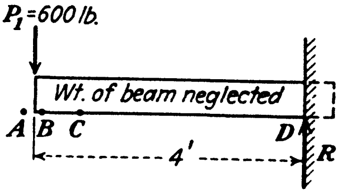


FIG. 72.

By the foregoing procedure, the reaction R is equal to the sum of the downward forces. In Fig. 72,

$$R = 600 \text{ lb.}$$

The shear at any section of the beam is the sum of the vertical forces to the *left* of the section. If we consider the section A as being just to the left of the left end of the beam, the sum of the forces to the left of section A is zero. Considering section B just to the right of the left end of the beam, the sum of all of the forces on the left now is equal to $P_1 = 600$ lb. *downward.*

At sections C or D the sum of the forces on the left is still equal to 600 lb. downward. It is now possible to draw a shear diagram as shown in Fig. 73. As the directions of the shears at sections B, C, and D are downward, they are plotted below a horizontal zero shear line (0-0).

Figure 74 shows a cantilever beam with a uniform load and two concentrated loads. Determining the amount of the reaction, $R = 400 + 500 + 6 \times 50 = 1200$ lb.

The shear at the left end is zero. At the section just to the left of P_1 the sum of the loads to the left is $1 \times 50 = 50$ lb. downward. At the section just to the right of P_1, the sum of the loads is $P_1 + 1 \times 50 = 400 + 50 = 450$ lb. downward. At the section just to the left of P_2 the sum of the loads is

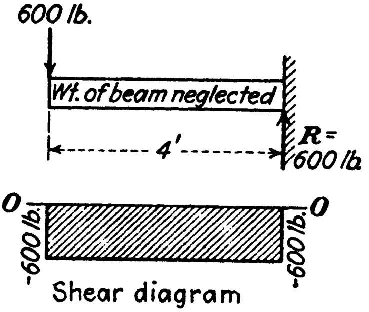


FIG. 73.

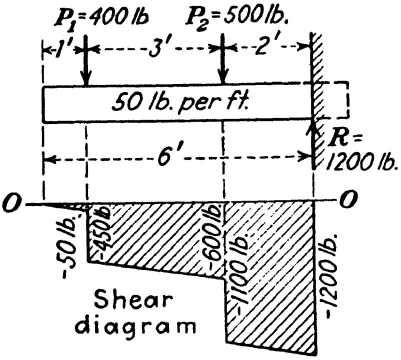


FIG. 74.

$$P_1 + 4 \times 50 = 400 + 200 = 600 \text{ lb.}$$

downward. At the section just to the right of P_2, the sum of the loads is $P_1 + P_2 + 4 \times 50 = 400 + 500 + 200 = 1100$ lb. downward. At the section just to the left of the wall, the sum of the loads is $P_1 + P_2 + 6 \times 50 = 400 + 500 + 300 = 1200$ lb. downward. Drawing the shear diagram in Fig. 74, it will be seen that the diagram is entirely below the zero-horizontal shear line marked 0-0.

It should be noticed that the shear diagrams in Figs. 73 and 74 do not change signs at any section, but the shear at the section at the wall is zero. This is an important section as will be noted later.

Problems

68. Draw the shear diagram for a simple beam 20 ft. long with a single concentrated load of 650 lb. applied 12 ft. from the left end. Neglect the weight of the beam.

69. A beam 12 ft. long carries a uniformly distributed load of 120 lb. per ft. Draw the shear diagram, neglecting the weight of the beam.

70. In problem 69, assume that an additional force P of 500 lb. acts downward at the middle of the beam. Draw the shear diagram.

71. A simple beam 24 ft. long weighs 100 lb. per ft. of length. Draw the shear diagram when concentrated loads of 800 lb. and 50 lb. are applied at 14 ft. and 20 ft., respectively, from the left end of the beam.

72. Draw the shear diagram for Fig. 75.

73. Draw the shear diagram for a cantilever beam 8 ft. long, weighing 100 lb. per ft. with a downward concentrated load of 800 lb. applied 3 ft. from the free end.

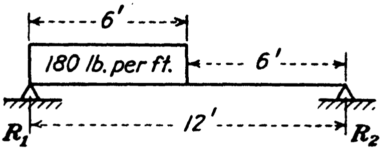


FIG. 75.

Bending Moments.—The principle of moments may now be used to find the amount of bending, or, as we say, the bending moment, for any section in the beam. The bending moment for any section in a beam is defined as the algebraic sum of all the moments of the forces acting on *either* side of the section. The moments of the forces on *either* side may be taken because, as the beam is in equilibrium, they will be the same. However, for the sake of uniformity, the left side is always used.

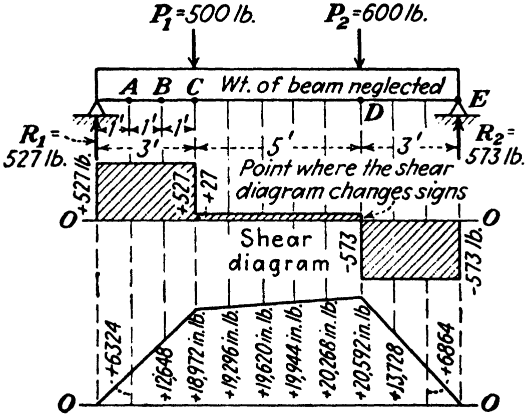


FIG. 76.

In calculating the bending moment for any point or section, the moment arm should always be expressed in *inches* and the force in *pounds*. Then the bending moment will be in *inch-pounds*.

Figure 70 has been recopied as Fig. 76, with the addition of the bending moment diagram, which is calculated as follows: considering all the beam to the left of section A, as shown in Fig. 77, we have only the reaction R_1 acting on the 1-ft. piece of beam. Now, take the moments about the point (section) A with the lever arm in inches and representing the moment at A by M_A:

$$M_A = 527 \text{ lb.} \times 12 \text{ in.} = 6324 \text{ in.-lb.}$$

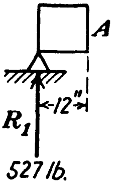


FIG. 77.

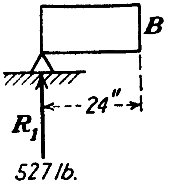


FIG. 78.

As the moment is clockwise, it has a positive sign, and hence

$$M_A = +6324 \text{ in.-lb.}$$

Considering all the beam to the *left* of section B, as shown in Fig. 78, and taking the moments about the point (section) B:

$$M_B = 527 \text{ lb.} \times 24 \text{ in.} = +12{,}648 \text{ in.-lb.}$$

Considering all the beam to the *left* of section C, as shown in Fig. 79, and taking moments about the point (section) C:

$$M_C = 527 \text{ lb.} \times 36 \text{ in.} - 500 \text{ lb.} \times 0 = +18{,}972 \text{ in.-lb.}$$

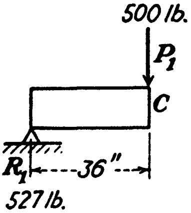


FIG. 79.

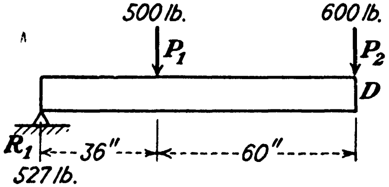


FIG. 80.

Considering all the beam to the *left* of section D as shown in Fig. 80, and taking moments about the point (section) D:

$$\begin{aligned} M_D &= +527 \text{ lb.} \times 96 \text{ in.} - 500 \text{ lb.} \times 60 \text{ in.} - 600 \text{ lb.} \times 0 \\ &= +50{,}592 - 30{,}000 - 0 = +20{,}592 \text{ in.-lb.} \end{aligned}$$

Considering all the beam to the *left* of section E as shown in Fig. 76, and taking moments about the point (section) E:

$$
\begin{aligned}
M_E &= +527 \text{ lb.} \times 132 \text{ in.} - 500 \text{ lb.} \times 96 \text{ in.} - 600 \text{ lb.} \times 36 \text{ in.} + 573 \times 0 \\
&= +69{,}564 - 48{,}000 - 21{,}600 + 0 \\
&= 69{,}564 - 69{,}600 \\
&= 0
\end{aligned}
$$

Note: These should be equal, but due to the failure to use the decimal value of $R_1 = 527.27$ lb., the numerical moments appear incorrect.

Plotting the bending moments as obtained for sections *A*, *B*, *C*, *D*, and *E* on Fig. 76, we connect the points plotted and obtain a **bending moment diagram,** as shown.

It should be noted that the *bending moment diagram consists of straight lines.* This is the case where the beam supports concentrated loads only. It is not generally necessary to calculate the bending moment at sections one foot apart, but only as those sections where the shear diagram shows an abrupt change as at *C* and *D*. However, as is explained later, the bending moment must be calculated for the section where the shear diagram changes sign as at section *D* in this example.

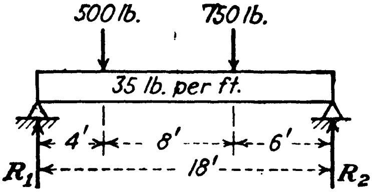


FIG. 81.

Figure 81 shows a beam with two concentrated loads and a uniform load of 35 lb. per ft. Let it be required to calculate the bending moment for various sections of the beam.

It will first be necessary to solve for the reactions R_1 and R_2 in order that all the forces on the beam will be known Taking moments about R_1 we get

$$500 \times 4 + 750 \times 12 + 35 \times 18 \times 9 = R_2 \times 18$$

from which

$$R_2 = 926 \text{ lb.}$$

Likewise, taking moments about R_2 we get

$$750 \times 6 - 35 \times 18 \times 9 - 500 \times 14 = R_1 \times 18$$

from which

$$R_1 = 954 \text{ lb.}$$

Now let us calculate the bending moment at a point A, which is 2 ft. from the left end of the beam, as shown in Fig. 82. There are but two forces acting to the left of A so that these are the only forces which must be considered. These forces are the reaction R_1 and the weight of 2 ft. of the beam acting down. This weight of the 2 ft. of the beam will be 2 times 35 or 70 lb., and, therefore, may be considered as a force of 70 lb. acting in the middle of this section.

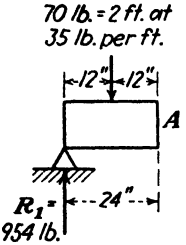


FIG. 82.

Now take moments about the point A and remember that clockwise moments are positive and counter-clockwise moments are negative. Also remember that the moment arms should be in inches. By representing the moment about A by M_A,

$$M_A = 954 \times 24 - 70 \times 12 = +22{,}056 \text{ in.-lb.}$$

Likewise, let us find the bending moment for a point B, 10 ft. from the left end, as shown in Fig. 83. There will now be three forces to consider, namely, the reaction R_1, the uniform load and the force of 500 lb. acting 4 ft. from R_1. Expressing the moment at B by M_B, we get

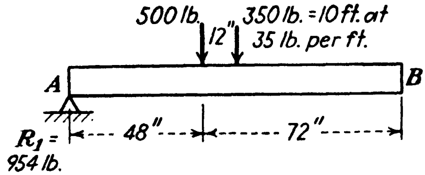


FIG. 83.

$$M_B = (954 \times 10 \times 12) - (500 \times 6 \times 12) - (35 \times 10 \times 5 \times 12)$$

Each term has been multiplied by 12 to reduce the moment arms to inches. In the last term 35 times 10 gave the amount of the uniform load, the 5 was the lever arm since the load may be considered as being concentrated in the middle of the section under consideration, while the 12 reduced this moment arm to inches. Solving the equation, we get

$$M_B = +57{,}480 \text{ in.-lb.}$$

Thus, it will be seen that the bending moment at B is considerably larger than at A.

Now it is perfectly natural that in designing a beam it will be necessary to find the section of maximum bending moment which will be the section of maximum stress. To do this a curve might be plotted as shown in Fig. 84. This curve has been obtained by solving for the bending moment at various points along this beam and then marking the values up on vertical lines throughout the section where the value was calculated.

The curve shows that the maximum bending moment is equal to 59,136 in.-lb. and that it comes just where the load of 750 lb. is applied. Such a curve or graph is called a bending moment diagram.

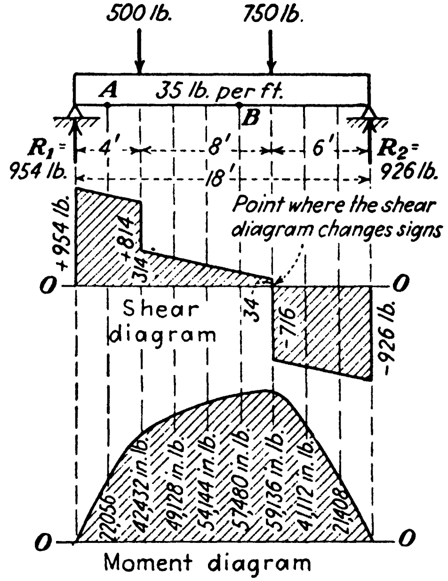


FIG. 84.

If the beam supports a uniform load (with or without concentrated loads) the *bending moment diagram will be curved* as shown in Fig. 84. Now, it will be seen that the **maximum bending moment** in Fig. 76 is 20,592 in.-lb. and that it occurs at the same section of the beam where the shear diagram changes from positive to negative. This is not a coincidence but is an established rule, namely, that the *bending moment is maximum where the shear changes sign.* In Fig. 84, the shear diagram changes sign under the 750-lb. load, and the maximum bending moment of 59,136 in.-lb. occurs at this section.

It is, therefore, essential to draw the shear diagram for every beam to locate the section where this diagram changes sign.

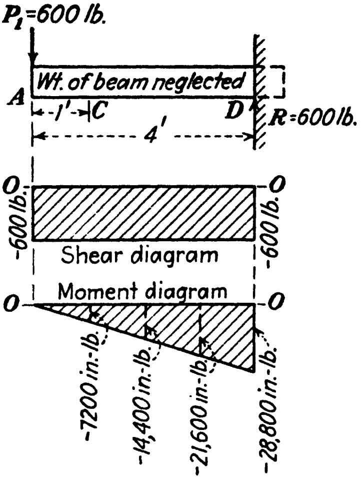


Fig. 85.

As the wall furnishes the sole support for a cantilever type beam, the calculations of the moments at the various sections must be made by using only the portion of the beam, to the left of the section.

Figure 73 has been recopied as Fig. 85. The moment at the end *A* of a cantilever beam is zero as there are no forces to the left of this section.

Considering all the beam to the left of section *C* (1 ft. from *A*) as shown in Fig. 86, and taking moments about the point (section) *C*:

$$M_c = 600 \times 12 = 7200 \text{ in.-lb.}$$

As the moment is counter-clockwise, it is negative, hence

$$M_c = -7200 \text{ in.-lb.}$$

Considering all the beam to the left of section *D* (6 ft. from *A*) as shown in Fig. 87, and taking moments about the point (section) *D*:

$$M_D = -600 \times 48 = -28{,}800 \text{ in.-lb.}$$

Similarly, the moments at sections 2 ft. and 3 ft. from the left end *A* can be calculated. These are equal to −14,000 in.-lb. and −21,600 in.-lb.

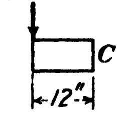


Fig. 86.

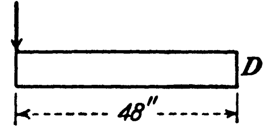


Fig. 87.

The values of the moments can be plotted as shown in Fig. 85, where it will be seen that the points fall on a straight line. The maximum bending moment occurs at the wall section, as will be noted. This is always the case with cantilever beams.

Figure 74 has been recopied as Fig. 88, and the moments calculated for sections B, C, and D as shown in the Figs. 89, 90, and 91.

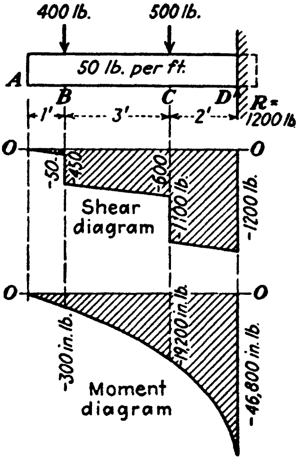


FIG. 88.

The moment at the end A of the cantilever beam is zero as there are no forces to the left of this section.

With section B (1 ft. from A) as the center of moments, considering all forces to the left of section B as shown in Fig. 89:

$$M_B = -50 \times 1 \times 6 - 400 \times 0 = -300 \text{ in.-lb.}$$

which is counter-clockwise or negative.

With section C (4 ft. from A) as the center of moments, considering all forces to the left of section C as shown in Fig. 90:

$$M_c = -50 \times 4 \times 24 - 400 \times 36 - 500 \times 0 = -19{,}200 \text{ in.-lb.}$$

Similarly, the sum of the moments about point (section) D, as shown in Fig. 91, is

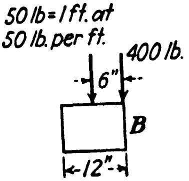


FIG. 89.

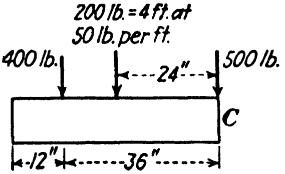


FIG. 90.

$$M_D = -50 \times 6 \times 36 - 400 \times 60 - 500 \times 24$$
$$= -46{,}800 \text{ in.-lb.}$$

The line connecting the plotted bending moments as calculated above will be seen in Fig. 88. As the beam is uniformly loaded, the diagram of the bending moments is curved. Like the maximum bending moment in Fig. 85, the maximum bending moment for the cantilever beam of Fig. 88 occurs at the wall.

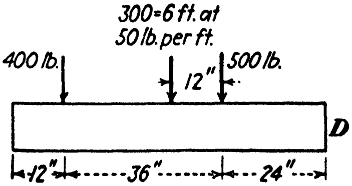


FIG. 91.

Problems

74. Draw the shear and bending moment diagrams for a simple beam 20 ft. long, supporting a concentrated load of 200 lb. at 12 ft. from the left end.

75. A simple beam 24 ft. long weighs 100 lb. per ft. of length. It supports concentrated loads of 800 lb. at 14 ft. from the left end and 50 lb. at 4 ft. from the right end. Draw the shear and moment diagrams for this beam.

76. Draw the shear and moment diagrams for Fig. 75.

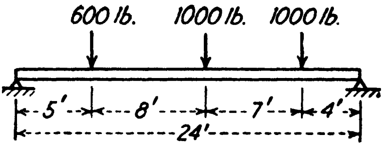


FIG. 92.

77. Draw the shear and moment diagrams for Fig. 92.

78. Draw the shear and moment diagrams for a cantilever beam 8 ft. long weighing 100 lb. per ft. of length which supports a concentrated load of 800 lb. at 3 ft. from the free end.

CHAPTER VII

STRESSES IN BEAMS

In designing a beam it will be found that its strength depends more upon its shape than upon the actual amount of metal or wood in it. Most beams of steel are the shape of the I-beam or the channel, both of which are illustrated in Fig. 93. In either of these, it will be seen that the actual cross section area will not be very great.

If the I-beam or the channel had been rolled out in the form of a round steel bar with the same cross section area, it would appear as in Fig. 94. In either case there would be the same amount of metal in the beam. However, the I-beam and the channel each are many units stronger in bending than the same amount of metal when put in a compact mass such as in Fig. 94. Hence, the shape and the arrangement of the metal, and not the cross section area, do most to determine the resistance to bending.

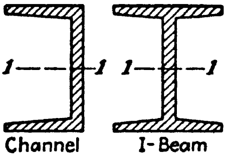


FIG. 93.

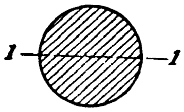


FIG. 94.

The neutral axis has been defined as the plane at which there is no deformation and it follows directly that there is no stress on this plane. It can be shown that the **neutral axis** of any beam coincides with the **center of gravity,** and as we work only with the cross section area, the neutral axis and the center of gravity of the cross section area are the same line.

General Formula for the Design of Beams.—The general formula for the design of beams may now be written

$$M = S\frac{I}{c}$$

in which M is the maximum bending moment in *inch-pounds*.

S = allowable bending stress of the material

Note: Bending stress in any beam is tension in those layers which are stretched, and compression in those layers which are shortened.

I = moment of inertia of the cross section about the neutral axis

c = distance from the neutral axis to the top or bottom of the cross section, whichever is the larger, or at which ever point the stress is desired

(If the general formula is solved for S, or $S = Mc/I$ it will be noted that the bending stress S increases directly as the distance c increases.)

The ultimate strength and the factor of safety of materials can be found from tables in the Appendix, or the allowable stress will be specified. Also, the maximum bending moment can be found as has been done in the last few pages. Knowing these two, therefore, I/c, which is called the **section modulus**, can be found by solving the equation $M = S\frac{I}{c}$.

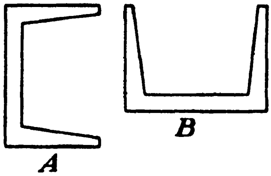


FIG. 95.

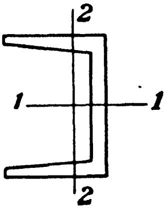


FIG. 96.

Values of I/c for I-beams, channels, angles, etc., are given in the tables in the Appendix under the column heading S. The student is cautioned against being confused by the symbol S for the section modulus as given in the tables. It is the symbol used for the value of I/c.

Since most beams can be used in two general positions, the tables give the section modulus for the two positions. For example, a channel might be used in either of the positions shown in Fig. 95. On this account the tables show an illustration as in Fig. 96, with the lines 1-1 and 2-2 drawn through them, these lines meaning the neutral axis for the two positions. If the beam is to be used standing up, as at A, Fig. 95, the section modulus 1-1 will be used, but if the beam is to be used as at B, Fig. 95, then the section modulus for the axis 2-2 will be used.

The American Institute of Steel Construction specifies that the allowable bending stress for steel beams is 20,000 lb. per sq. in. The most recent studies on structural timber have been made by the Forest Products Laboratory, U. S. Forest Service, and have been incorporated into the specifications of the American Society for Testing Materials. The recommendations for the various species of timber under the type of service to which the beam is subjected vary considerably, but a safe working stress may be assumed to be 1000 lb. per sq. in.

The Appendix, under the heading Elements of Sections (formulas for calculations), gives the formula for the section modulus as S. The usual form of a timber beam has a rectangular cross section with the axis of moments through the center.

Let it be required to calculate the height of a rectangular timber beam 2 in. wide to withstand safely the maximum bending moment for the beam in Fig. 76.

$$\text{Maximum bending moment} = 20{,}592 \text{ in.-lb.}$$

Allowable unit bending stress for timber = 1000 lb. per sq. in.

Substituting these values in the equation $M = S\frac{I}{c}$ we have

$$20{,}592 = \frac{I}{c} \times 1000$$

from which

$$\frac{I}{c} = \frac{20{,}592}{1{,}000} = 20.592 \text{ in.}^3$$

From the Appendix (Elements of Sections), for a rectangular beam with the axis of moments through the center,

$$\frac{I}{c} = S = \frac{bd^2}{6} \qquad \text{where} \qquad d = \text{height}$$

and in which

$$b = 2 \text{ in. (given width)} \qquad \text{and} \qquad \frac{I}{c} = 20.592 \text{ in.}^3$$

Then

$$20.592 = \frac{2d^2}{6}$$

and

$$d^2 = \frac{20.592 \times 6}{2} = 61.776$$

$$d = \sqrt{61.776} = 7.86 \text{ in.} \quad \text{Use 8-in. height.}$$

If a steel I-beam or channel is to be used to support the loads as shown in Fig. 76, the safe allowable unit bending stress for steel is 20,000 lb. per sq. in.

Substituting in the equation:

$$M = S\frac{I}{c}$$

$$20{,}592 = \frac{I}{c} \times 20{,}000$$

$$\frac{I}{c} = \frac{20{,}592}{20{,}000} = 1.030 \text{ in.}^3$$

Let us assume that we want an I-beam and will use it in an upright position, as shown in Fig. 97. Then we will want the section modulus about axis 1-1. These values for the section modulus about the axis 1-1 can be found by referring to the Appendix.

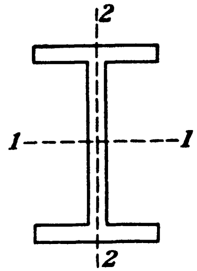


FIG. 97.

Due to the large number of sizes of I-beams manufactured by the different steel companies, the preceding and following examples involving selections of steel beams are based on the American Standard Sections. Thus, by referring to the Appendix, these sections are available to the student from which to make a selection of the proper size. However, the selection from any list of beams is made on the same basis whether the beam sections are wide flange sections (W) or American Standard made by the Bethlehem, United States Steel, or any other company.

Hence, in all problems, involving steel beams, the **beams, American Standard,** will be used.

If we want a channel instead of an I-beam, then we refer to the list of channels given for a section modulus equal to or greater than 1.030. We find this section or channel to be 3 in. deep weighing 4.1 lb. per ft.

The design of angle irons and other standard shapes will follow the same order. After knowing the section modulus required, the student may refer to the pages in the Appendix which give values for the type of beam he desires and select the proper one.

As 1.030 is the required section modulus of the beam, we can look down the list of I-beams and find one which has a section modulus *equal to or greater than* the required value. It will be found that the smallest and lightest, 3-in. I at 5.7 lb. per ft. has a section modulus of 1.7 about the 1-1 axis, and this size will be selected.

Referring to Fig. 88, the maximum bending moment in the cantilever beam is −46,800 in.-lb. The negative sign is disregarded in selecting the correct size beam to safely support the loads, hence in the following calculations, it will not be written.

Select a steel channel to safely support the loads on the beam shown in Fig. 88.

$$M = S\frac{I}{c}$$

then

$$46{,}800 = \frac{I}{c} \times 20{,}000$$

therefore

$$\frac{I}{c} = \frac{46{,}800}{20{,}000} = 2.34$$

Referring to the Appendix, it will be seen that a 5-in. 6.7-lb. per ft. steel channel will provide a section modulus greater than 2.34 as required.

Steel is purchased on the basis of the cost per pound, so where there are no limiting factors other than the economical design, the lightest weight beam having the required section modulus should always be selected.

Problems

In each of the following problems calculate the reactions; draw the shear diagram; locate the point where the shear diagram changes sign; determine the maximum moment. Select a beam as noted.

79. What is the unit bending stress in a 4-in. × 12-in. timber beam when supporting a concentrated load as shown in Fig. 98?

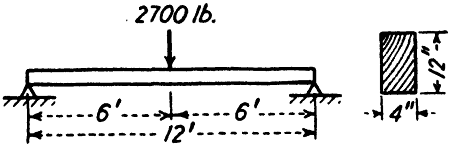


Fig. 98.

80. What width *b* of a timber beam 12 in. deep is required to support the uniform load shown in Fig. 99? The allowable unit bending stress = 1000 lb. per sq. in.

81. Using the allowable bending stress as specified by the American Institute of Steel Construction, what section modulus is necessary for a steel I-beam to support the loads in Fig. 100?

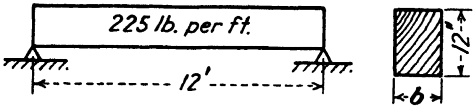


FIG. 99.

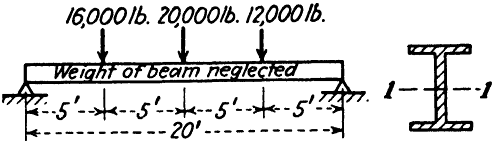


FIG. 100.

82. What size I-beam is required to support the loads in problem 75? The selection should be made to satisfy the required section modulus, and at the same time to have the least weight per foot. The latter item is the basis of economical design.

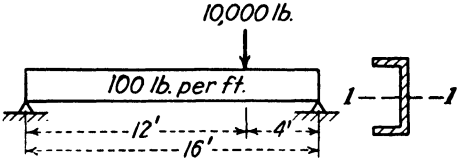


FIG. 101

83. Select the most economical steel channel to support the loads shown in Fig. 101.

84. A steel I-beam is used as the cantilever beam of problem 78. What size beam should be used?

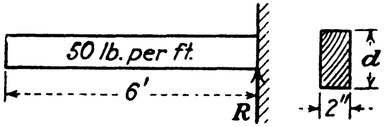


FIG. 102.

85. A timber beam 2 in. wide is used for the cantilever beam in Fig. 102. What height (*d*) should be used?

Shearing Stress in Beams.—If we were to use a number of 2-in. × 6-in. planks as a beam, Fig. 103, considering them weightless, and then apply a vertical load to the beam as shown in Fig. 104, we would find that the adjacent faces of any two of the planks would slide or shear with respect to each other. This is particularly true if the beam is made of timber, where the material has natural planes of cleavage.

Considering a very small piece of the beam A in Fig. 103, the unit shearing stress on this piece is equal in either a horizontal or a vertical direction. Thus

$$S_v = S_h$$

Horizontal Shearing Stress.—The formula by which the unit horizontal shearing stress (and vertical shearing stress) is calculated is derived by a process involving considerable advanced mathematics, so it will suffice to give the formula and define the meaning of the terms involved.

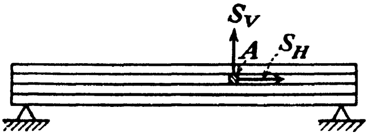


FIG. 103.

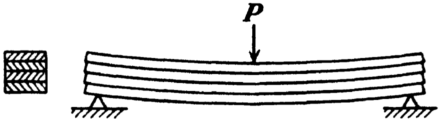


FIG. 104.

$$S_h = \frac{V}{It}a'\bar{y}$$

where S_h = maximum unit horizontal shearing stress in the beam (pounds per square inch)
V = maximum total vertical shear in the beam (pounds). Obtained from the shear diagram
I = moment of inertia of the cross-section of the beam about the neutral axis (center of gravity)
t = width of the beam at the *neutral axis*
a' = area of the beam above or below the neutral axis
$\bar{y}$ = distance from the *neutral axis* to the *center of gravity* of the a' area

Example.—A timber beam 4 in. × 6 in. 10 ft. long is used to support a concentrated load of 3000 lb. at the center of the beam. The beam is supported at the ends. What is the maximum unit horizontal shearing stress in the beam?

By plotting a shear diagram, it will be seen that the maximum value of the total vertical shear V is 1500 lb.

Then:

$V = 1500$ lb.
$I = \frac{1}{12}bh^3 = \frac{1}{12}4(6)^3 = 72$
$t = 4$ in.
a' = area above the neutral axis (Fig. 105)
$= 4 \times 3 = 12$ sq. in.

$\bar{y}$ = distance from the neutral axis to the center of gravity of the area $= \frac{1}{2}(3) = 1\frac{1}{2}$ in.

$$S_h = \frac{V}{It} a'\bar{y}$$

$$S_h = \frac{1500}{72 \times 4} (12)\left(1\frac{1}{2}\right)$$

$$= \frac{1500}{16} = 93.75 \text{ lb. per sq. in.}$$

(As $S_h = S_v$ it follows that the maximum unit vertical shear is 93.75 lb. per sq. in. That is, the beam has equal horizontal and vertical shear stresses (Fig. 103). However, the timber is easier to shear along the grain which is normally placed horizontally.)

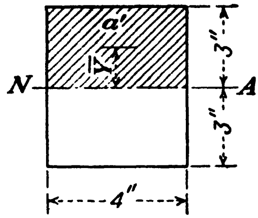


FIG. 105.

It is often customary in beams of *rectangular cross sections* to calculate the maximum unit horizontal shear by reducing the full equation to

$$S_h = \frac{V}{It} a'\bar{y}$$

$$= \frac{V}{\frac{bh^3}{12}(b)}\left(b\frac{h}{2}\right)\left(\frac{h}{4}\right)$$

$$= \frac{3}{2}\frac{V}{bh}$$

The term V/bh represents the *average* unit vertical shear in the beam, hence $\frac{3}{2}$ of the average unit vertical shear in a beam of *rectangular cross section* equals the *maximum unit horizontal shear*

Applying this to the preceding example:

$$V = 1500 \text{ lb.}$$

$$bh = 4 \times 6 = 24 \text{ sq. in.}$$

$$S_h = \frac{3}{2}\frac{V}{bh} = \frac{3 \times 1500}{2 \times 24} = \frac{1500}{16} = 93.75 \text{ lb. per sq. in.}$$

I-Beam Shearing Stress.—The usual assumption made in the case of steel I-beams, channels and T-beams is that the total vertical shear is carried entirely by the web of the beam, the web extending the *full height* of the beam. While the flanges offer some little resistance to vertical shear, the preceding assumption is on the side of safety. Thus, the *average* vertical web shear

$$\text{Average } S_v = \frac{V}{\text{web area}} = \frac{V}{td}$$

where t is the thickness of the web, and d is the *full* height of the beam.

Example.—A cantilever beam 6 ft. long is to support a uniform load of 4000 lb. per ft. What size steel I-beam is required? Is the web safe if the allowable average vertical shear is 13,000 lb. per sq. in. as specified by the American Institute of Steel Construction?

By drawing the shear and moment diagrams, maximum $V = 24{,}000$ lb., and the maximum bending moment $M = 72{,}000$ ft.-lb. Using an allowable bending stress = 20,000 lb. per sq. in.

$$M = S\frac{I}{c}$$

$$72{,}000 \times 12 = 20{,}000\frac{I}{c}$$

$$\frac{I}{c} = 43.2 \text{ in.}^3 \text{ required section modulus}$$

From the table of American Standard I-beams, the lightest I-beam with a section modulus greater than 43.2 is a 12-in. I-40.8 lb. per ft. The web thickness of this I-beam is 0.460 in. and the depth is 12 in. The total web area is 0.460 × 12 in. Then

$$\text{Average } S_v = \frac{V}{td} = \frac{24{,}000}{0.460 \times 12} = 4350 \text{ lb. per sq. in.}$$

As this is less than the safe allowable average unit vertical shear for steel of 13,000 lb. per sq. in., the beam is safe.

Problems

86 In problem 68, if the timber beam has a width of 2 in. and a height of 4 in., and the safe allowable shearing stress is 100 lb. per sq. in., is the beam safe in horizontal shear?

87. In problem 69, what is the maximum unit horizontal shear in a timber beam 2 in. wide × 6 in. in depth?

88. In problem 71, the maximum allowable unit horizontal shear for timber is 100 lb. per sq. in. What size piece of timber is necessary if timber 2 in. thick is available? Note that timber sizes vary in 2-in. steps: 2 in. × 4 in.; 2 in. × 6 in.; 2 in. × 8 in., etc.

89. In problem 73, is a 4-in. × 6-in. timber beam safe to resist a maximum horizontal shear of 100 lb. per sq. in.?

90. A beam of timber is 10 ft. long and has a width of 4 in. The beam is supported at the ends and supports a uniform load of 200 lb. per lineal ft. as well as a concentrated load of 1000 lb. on the center. What *actual* height of beam is required using the Forest Products Laboratory safe bending stress? What maximum unit horizontal shearing stress is developed in this size beam?

91. A beam 3 ft. long supports 100,000 lb. per ft. If the beam rests on reactions at the ends, what is the maximum average unit vertical shear in the web of a 15-in.-55 lb. per ft. American Standard I-beam? Would this beam be considered safe according to the specified allowable web shearing stress given in the preceding example?

92. An American Standard 10-in. channel weighing 30 lb. per ft. can withstand what total vertical shear if the allowable average web shear is 13,000 lb. per sq. in.?

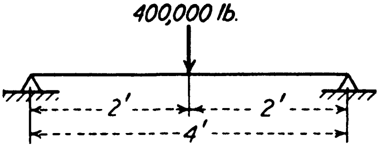


FIG. 106.

93. Select the proper size I-beam and check it for an average vertical web shear stress of 13,000 lb. per sq. in. for the beam shown in Fig. 106.

CHAPTER VIII

BEAM DEFLECTIONS

When a beam is supporting a load or system of loads, the beam sags or, as the engineer says, is deflected. All parts of the beam, except the sections of the reactions, move downward varying distances. At one section of the beam, the deflection or sag is a maximum, and it is the maximum deflection in which we are interested.

Figure 107 shows the initial straight position of a beam where the weight of the beam is neglected. The neutral axis AB is a plane. Figure 108 shows the same beam with a single concentrated load applied, and it will be noted that the neutral axis AB is now a curved surface. The curve which the neutral axis assumes when the beam is loaded is called the **elastic curve,** and the deflection in a beam is always measured with reference to the straight position of the elastic curve. We might define the deflection of a beam as the vertical displacement of the neutral axis.

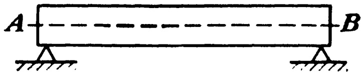


Fig. 107.

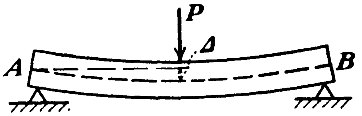


Fig. 108.

Deflections in beams have considerable importance in the design of beams because excessive deflections may cause serious trouble where a level floor is desirable or where the plaster on the underside of the floor joist may crack.

The specified limiting value of the maximum deflection of a beam is $\frac{1}{360}$ of the span (length) of the beam.

Maximum Deflections for Beams.—As the derivations of the maximum deflections for beams under various systems of loading are beyond the scope of simple mathematics, the formulas for the deflections will be given.

1. Cantilever beam with a concentrated load on the free end:

$$\Delta = \frac{PL^3}{3EI}$$

2. Cantilever beam supporting a total uniform load of W:

$$\Delta = \frac{WL^3}{8EI}$$

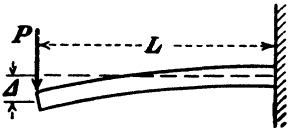


FIG. 109.

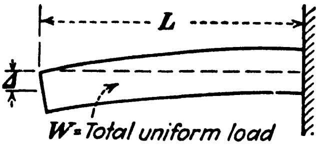


FIG. 110.

3. Simple beam supported at the ends, with a concentrated load P at the center:

$$\Delta = \frac{PL^3}{48EI}$$

4. Simple beam supporting a total uniform load W:

$$\Delta = \frac{5WL^3}{384EI}$$

By the principle of **superposition**, the total deflection caused by a combination of loads acting on a beam can be obtained by adding the deflections due to the separate loads, provided the elastic (limit) deflection of the beam has not been exceeded.

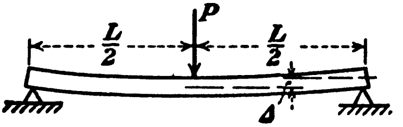


FIG. 111.

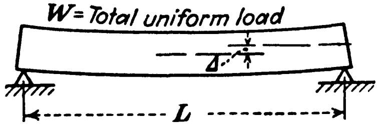


FIG. 112.

Example.—A timber cantilever beam 2 in. × 12 in. × 6 ft. long is used to support a uniform load of 100 lb. per ft.

$$E = 1{,}200{,}000 \text{ lb. per sq. in.}$$

What is the maximum deflection?

$$W = \text{total uniform load} = 6 \times 100 = 600 \text{ lb.}$$

By equation for case 2

$$\Delta = \frac{WL^3}{8EI}$$

where L = length in inches
I = moment of inertia of the cross section about the center of gravity

$$\Delta = \frac{600 \times (72)^3}{8 \times 1{,}200{,}000 \times \frac{2(12)^3}{12}} = \frac{81}{1000} = 0.081 \text{ in.}$$

The maximum allowable deflection is $\frac{1}{360}$ of the span which is $\frac{1}{360}$ × (length in inches):

$$\text{Allowable } \Delta = \tfrac{1}{360}(72) = 0.20 \text{ in.}$$

As the actual Δ is 0.081 in. and the allowable $\Delta = 0.20$ in., this beam is satisfactory as far as the limiting deflection is concerned.

Example.—A simple beam 2 in. × 12 in. × 10 ft. long is to support a single concentrated load of 1000 lb. at the center. The bending stress shall not exceed 1000 lb. per sq. in., the maximum horizontal shearing stress shall not exceed 100 lb. per sq. in., the maximum deflection shall be less than the prescribed limits Is the beam safe?

$$E = 1{,}200{,}000 \text{ lb. per sq. in.}$$

From the shear diagram, maximum $V = 500$ lb., and from the moment diagram, maximum $M = 30{,}000$ in.-lb.

$$M = S\frac{I}{c} \qquad \text{and} \qquad S = \frac{Mc}{I}$$

$$= \frac{30{,}000 \times 6}{\frac{2(12)^3}{12}} = 625 \text{ lb. per sq. in.}$$

Beam is safe in bending as 625 lb. per sq. in. is less than the allowable stress.

$$S_h = \frac{3}{2}\frac{V}{bh} = \frac{3 \times 500}{2 \times 2 \times 12} = 31.25 \text{ lb. per sq. in.}$$

The beam is safe in horizontal shear.

By equation for case 3

$$\Delta = \frac{PL^3}{48EI}$$

$$\Delta = \frac{1000(120)^3}{48 \times 1{,}200{,}000 \times \frac{2 \times 12^3}{12}} = 0.104 \text{ in. actual deflection}$$

Allowable deflection $= \frac{1}{360}$ span $= \frac{1}{360} \times 120 = 0.333$ in.

Beam is safe as the actual deflection is less than the allowable.

Example.—A 12-in. I-beam weighing 45 lb. per ft. is to support a uniform load of 300 lb. per ft. which includes the weight of the beam, and a concentrated load of 2000 lb. at the center. The beam is 12 ft. long, supported at the ends. $E = 30{,}000{,}000$ lb. per sq. in. What is the total maximum deflection in the beam?

From the Appendix the moment of inertia of the 12-in. 45-lb. per ft. I-beam is 284.1 in.^{4}

By equation for case 3

$$\Delta = \frac{PL^3}{48EI}$$

$$\Delta = \frac{2000(144)^3}{48 \times 30{,}000{,}000 \times 284.1}$$

$\Delta = 0.0146$ in. due to the 2000-in. load.

By equation for case 4

$$\Delta = \frac{5WL^3}{384EI} \qquad \text{where} \qquad W = 300 \times 12 = 3600 \text{ lb.}$$

$$\Delta = \frac{5 \times 3600(144)^3}{384 \times 30{,}000{,}000 \times 284.1}$$

$\Delta = 0.0165$ in. due to the 300-lb.-per-ft. load.

By the principle of superposition, the total deflection is the sum of the partial deflections:

$$\begin{aligned}\text{Total } \Delta &= 0.0146 \text{ in.} + 0.0165 \text{ in.} \\ &= 0.0311 \text{ in.}\end{aligned}$$

Problems

94. A timber cantilever beam 8 ft. long is to support a uniform load of 250 lb. per ft. $E = 1{,}200{,}000$. What moment of inertia is required, so that the maximum deflection does not exceed the specified maximum?

95. A 6-ft. piece of steel I-beam is used as a cantilever to support a concentrated load of 5000 lb. at the free end, and a uniform load of 400 lb. per ft. $E = 30{,}000{,}000$. What size I-beam is necessary so that the maximum deflection is not excessive?

96. A 4-ft. beam having a circular cross section is supported at the ends and carries a concentrated load of 6000 lb. at the center. What size beam (radius) is required to be within the specified limits of deflection? $E = 30{,}000{,}000$.

97. A steel I-beam 18 ft. long is supported at the ends and is to carry 500 lb. per ft. Using an allowable bending stress of 20,000 lb. per sq. in., what size I-beam is necessary? What is the maximum deflection in this beam?

98. A diving board 2 in. × 12 in., laid flat, is 8 ft. long. $E = 1{,}500{,}000$. If a diver weighing 175 lb. stands on the end, how much does it sag?

99. An I-beam 30 ft. long is to support 400 lb. per ft. $E = 30{,}000{,}000$. If the beam is supported at the ends and the allowable bending stress is 20,000 lb. per sq. in., what size beam is required? Does this beam stay within the specified limits for maximum deflection? If not, what size I-beam would you need, and what bending stress is set up?

CHAPTER IX

COMBINED AXIAL AND BENDING STRESSES ECCENTRIC LOADS

In the previous work and discussions, the problems and examples dealt with stresses which were separately caused by axial, bending, or torsional loads. Many structural members are subjected to loads which cause combinations of these stresses, and the unit stress developed at any point may often be found by the principle of superposition. That is, if a member is subjected to two loads, one of which causes a tensile unit stress and the other a bending unit stress, the tensile unit stress may be calculated as though the bending unit stress were not present. Then the bending unit stress may be solved as though the tensile unit stress were not present. These unit stresses may then be combined as a resultant stress to obtain the actual stress provided that the elastic limit stress is not exceeded.

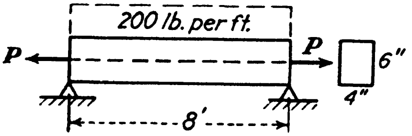


Fig. 113.

As each member and the applied loads must be analyzed, no single formula may be given. However, the fundamental formula for axial tension or compression

$$S = \frac{P}{A}$$

and the fundamental formula for bending stress in a beam

$$S = \frac{Mc}{I}$$

will be used in varying combinations of these stresses.

Example.—A timber beam (Fig. 113) is to support a uniform load of 200 lb. per ft. and an axial tensile load P of 2400 lb.

What is the maximum unit stress at the top and bottom of the beam?

As the maximum bending moment occurs at the center section of the beam, the amount of the bending moment obtained from a moment diagram is 19,200 in.-lb. When the beam sags, there will be tensile stress on the bottom A layer of the beam, and compressive stress on the top B layer of the beam. Calculating the unit bending stress at A and B, and calling it S_1 (Fig. 114)

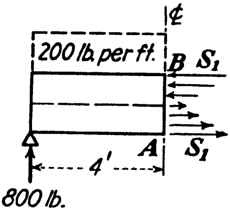


Fig. 114.

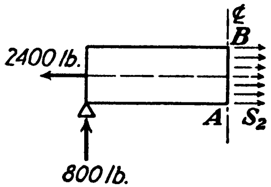


Fig. 115.

$$S_1 = \frac{Mc}{I}$$

$$S_1 = \frac{19{,}200 \times 3}{\frac{4(6)^3}{12}} = 800 \text{ lb. per sq. in.}$$

which is tension at A and compression at B.

If the bending stress is omitted, temporarily, and the stress as caused by the tensile force P is calculated, calling it S_2 (Fig. 115)

$$S_2 = \frac{P}{A}$$

$$= \frac{2400}{4 \times 6}$$

$$= 100 \text{ lb. per sq. in.}$$

This is a uniform tensile stress on any layer from A to B caused by the 2400-lb. force acting alone.

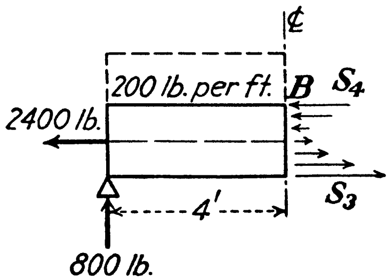


Fig. 116.

Now the two stresses at either A or B can be combined into a resultant (Fig. 116) by adding at A and subtracting at B. That is, we add the stresses algebraically, which gives S_3 as tension at A and S_4 as compression at B. Then these stresses, S_3 and S_4, will be the required maximum tensile and compressive stresses.

$S_3 = S_1 + S_2 = 800 + 100 = 900$ lb. per sq. in. tension.
$S_4 = S_1 - S_2 = 800 - 100 = 700$ lb. per sq. in. compression.

Example.—The frame of a small press is shown in Fig. 117. What unit stresses occur at A and B when the load P is 2400 lb.? I_x of the cross section is 120 in.^{4} and $A = 18$ sq. in.

To analyze the stresses which occur on the A-B cross section, it will be noted that the force P causes both bending and direct tension to occur. That is, the force P attempts to tear (bend) the frame so that it tears from A to B, and at the same time attempts to pull the frame apart on the AB cross section. Figure 118 shows the bending stress S_1 and the direct tensile stress S_2 as separate stress diagrams, although they really act simultaneously.

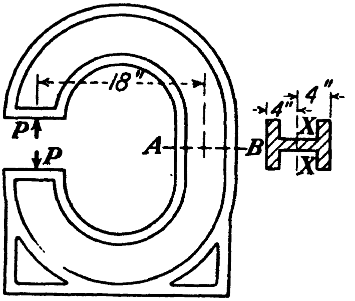


FIG. 117.

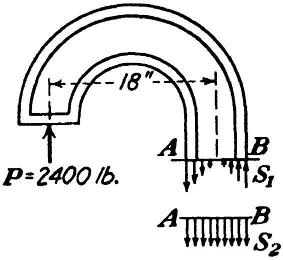


FIG. 118.

Calculating S_1, the moment of the force P about the neutral axis (center of gravity) of the cross section AB is

$$M = 2400 \times 18 = 43{,}200 \text{ in.-lb.}$$

$$S_1 = \frac{Mc}{I}$$

$$= \frac{43{,}200 \times 4}{120} = 1440 \text{ lb. per sq. in.}$$

which is tension at A and compression at B.

Calculating the direct tensile stress S_2

$$S_2 = \frac{P}{A} \quad \text{where} \quad A = 18 \text{ sq. in.}$$

$$S_2 = \tfrac{2400}{18} = 133.3 \text{ lb. per sq. in. tension at } A \text{ and } B$$

Then the resulting unit stress at A is

$$S_A = S_1 + S_2$$
$$= 1440 + 133.3$$
$$= 1573.3 \text{ lb. per sq. in. tension}$$

And the resulting unit stress at B is

$$S_B = S_1 - S_2 = 1440 - 133.3$$
$$= 1306.7 \text{ lb. per sq. in. compression}$$

Example.—The machine part (Fig. 119) is subjected to the force P equal to 2000 lb. Find the unit stresses at A and B.

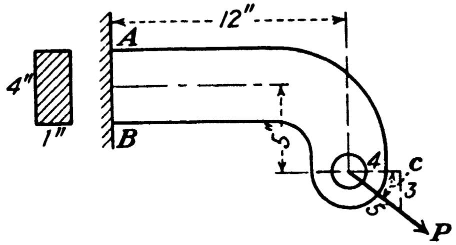


FIG. 119.

The load P may be resolved into its horizontal and vertical components:

$$H = P \cos c$$
$$= 2000 \times \tfrac{4}{5} = 1600 \text{ lb.}$$
$$V = P \sin c$$
$$= 2000 \times \tfrac{3}{5} = 1200 \text{ lb.}$$

The horizontal component causes bending at AB which places tension at B and compression at A for the member tends to tear from B to A. These stresses are designated S_1. This component also tends to pull the member directly away from the wall causing a tensile stress S_2, at A and B (Fig. 120). The vertical component causes bending at AB which gives tension at A and compression at B. These forces are designated S_3. Calculating the stresses S_1, S_2, and S_3 in order:

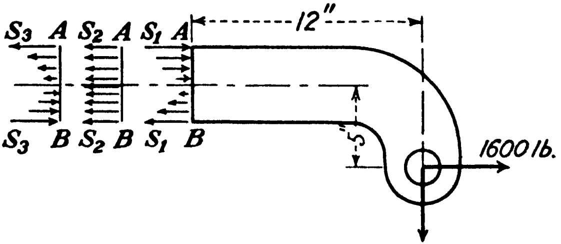


FIG. 120.

Moment of the 1600-lb. component about the center of gravity of the *AB* cross-section:

$$M = 1600 \times 5 = 8000 \text{ in.-lb.}$$

$$S_1 = \frac{Mc}{I}$$

$$= \frac{8000 \times 2}{\frac{1(4)^3}{12}} = 3000 \text{ lb. per sq. in. tension at } B \text{ and compression at } A$$

$$S_2 = \frac{P}{A}$$

$$S_2 = \frac{1600}{1 \times 4} = 400 \text{ lb. per sq. in. tension at } A \text{ and } B$$

Moment of the 1200-lb. component about the neutral axis of the *AB* cross section:

$$M = 1200 \times 12 = 14{,}400 \text{ in.-lb.}$$

$$S_3 = \frac{Mc}{I}$$

$$= \frac{14{,}400 \times 2}{\frac{1(4)^3}{12}} = 5400 \text{ lb. per sq. in. tension at } A \text{ and compression at } B$$

By referring to Fig. 120, it will be seen that the resultant stress at A will be the algebraic sum of S_1, S_2, and S_3 at A, or

$$S_A = -3000 + 400 + 5400 = 2800 \text{ lb. per sq. in. tension}$$

Note: As long as the kind of resulting stress is given, it is immaterial which algebraic sign is used to denote the separate tensile and compressive stresses as S_1, S_2, or S_3.

Then

$$S_B = 3000 + 400 - 5400 = -2000 \text{ lb. per sq. in. which is compression}$$

Eccentrically Loaded Riveted Joints.—Although it is desirable to have the line of action of the load pass through the center of gravity of the shearing areas of a system of rivets, it is not always possible. As a result, additional shearing stresses are set up in the rivets, which, if not considered in the design, may be the source of failure.

Figure 121 shows a plate to which is attached a force of 10,000 lb. riveted by four $\frac{3}{4}$-in. rivets to a horizontal plate E. The line of the load passes $1\frac{1}{2}$ in. to the right of the center of gravity of the four rivet areas. Due to the eccentric load, plate D tends to move straight down, and at the same time, rotate clockwise about the center of gravity C of the four rivet areas. Applying the principle of superposition, each tendency can be considered separately and the results combined.

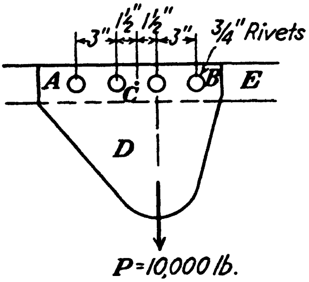


FIG. 121.

Considering the tendency to move in the direction of the applied force (straight down), four rivets will be sheared. Letting S_1 be the unit shearing stress (direct shear), we have

$$S_1 = \frac{P}{A} \text{ where } A \text{ is the total area of all rivets}$$

$$A = \left(\frac{\pi d^2}{4}\right)4 = 0.442 \times 4$$

$$= 1.768 \text{ sq. in.}$$

$$S_1 = \frac{10{,}000}{1.768} = 5660 \text{ lb. per sq. in.}$$

(Note that the arrows representing S_1 in Fig. 122 are resisting the force P.)

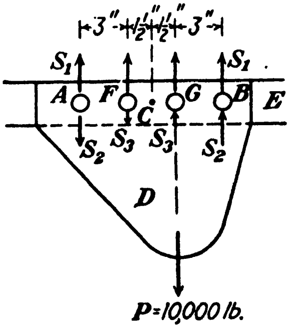


FIG. 122.

Letting S_2 represent the unit shearing stress due to the rotational trend about C, the total moment of the resisting shearing forces must be equal to the moment of the applied force. Now the resisting shearing force for any rivet must be perpendicular to a lever arm through the point C, and must produce a moment opposite in direction to the moment of P. Hence S_2 for rivet A must act at the center of this rivet and must produce counter-clockwise moment. Therefore it must be directed downward. Similarly S_2 for rivet B must act at the center of this rivet and must produce counter-clockwise moment, so it must be directed upward. The same idea would be true for the resisting stresses for rivets F and G. The moment of the resisting shearing stress for rivet A is the product of the force and the lever arm, or

$$\begin{aligned} M_A &= S_2 \times \text{area of the rivet} \times \text{lever arm} \\ &= S_2(0.442)4\tfrac{1}{2} \\ &= 1.983S_2 \end{aligned}$$

The moment of the resisting shearing stress for rivet B is the same as for rivet A, hence

$$M_B = 1.983S_2$$

For rivets F and G, the moments are each

$$M_F = M_G = S_3(0.442)1\tfrac{1}{2} = 0.663S_3$$

From the discussion on bending stresses in beams, it was noted that the stress was dependent upon the distance c from the center of gravity. This is likewise true in shafts or rotating members, and in riveted joints with an eccentric load. That is, the unit stress varies directly as the distance from the center of gravity. Then, if S_2 is $4\frac{1}{2}$ in. from the center of gravity, and S_3 is $1\frac{1}{2}$ in. from the center of gravity

$$\frac{S_2}{4\frac{1}{2}} = \frac{S_3}{1\frac{1}{2}}$$

and

$$S_3 = \frac{1\frac{1}{2}S_2}{4\frac{1}{2}} = \frac{1}{3}S_2$$

Thus

$$\begin{aligned} M_F &= M_G = 0.663S_3 \\ M_F &= M_G = (0.663)\tfrac{1}{3}S_2 = 0.221S_2 \end{aligned}$$

The sum of the resisting moments is the sum of the resisting moments of all rivets A, F, G, and B:

$$\begin{aligned} \text{Total } M &= (1.983S_2)2 + (0.221S_2)2 \\ &= 4.408S_2 \end{aligned}$$

But the resisting moment must equal the moment of the force (10,000 lb.) about C:

$$M = 10{,}000(1\tfrac{1}{2}) = 15{,}000 \text{ in.-lb.}$$

then

$$\begin{aligned} 4.408S_2 &= 15{,}000 \\ S_2 &= 3400 \text{ lb. per sq. in.} \end{aligned}$$

Referring to Fig. 122, at rivet A, S_1 and S_2 are in opposite directions, then

$$S_A = S_1 - S_2 = 5660 - 3400 = 2260 \text{ lb. per sq. in.}$$

and S_1 and S_2 are in the same direction on rivet B

$$S_B = S_1 + S_2 = 5660 + 3400 = 9060 \text{ lb. per sq. in.}$$

Example.—What is the shearing unit stress in rivets A and E in Fig. 123?

As in the previous example, the force P tends to produce two forms of motion of the plate F with respect to plate G: one to the right and the other rotation about point C. Letting S_1 represent the direct shearing stress to resist the motion to the right, Fig. 124,

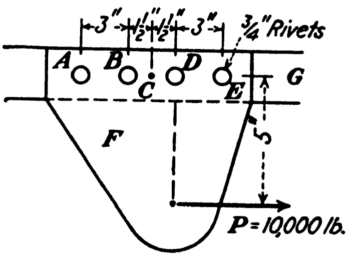


FIG. 123.

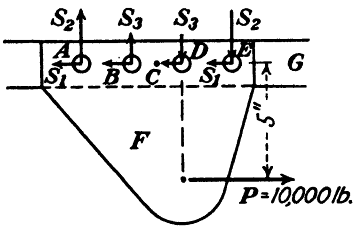


FIG. 124.

$$S_1 = \frac{P}{A}$$

$$= \frac{10{,}000}{4(0.442)} = 5660 \text{ lb. per sq. in. on each of } A,\ B,\ D, \text{ and } E.$$

The resisting moment for rivet A or E is

$$\begin{aligned} M_A = M_B &= (0.442S_2)4\tfrac{1}{2} \\ &= 1.983S_2 \end{aligned}$$

The resisting moment for rivet B or D is

$$\begin{aligned} M_B = M_D &= (0.442S_3)1\tfrac{1}{2} \\ &= 0.663S_3 \end{aligned}$$

But S_3 is to its distance from the center of gravity of the four rivets as S_2 is to its distance. Then

$$\frac{S_3}{1\frac{1}{2}} = \frac{S_2}{4\frac{1}{2}} \quad \text{and} \quad S_3 = \frac{1}{3}S_2$$

$$M_B = M_D = (0.663S_3) = (0.663)\tfrac{1}{3}S_2 = 0.221S_2$$

The total resisting moments of rivets A, B, D, and E will be

$$\begin{aligned} M &= 2(1.983S_2) + 2(0.221S_2) \\ &= 4.408S_2 \end{aligned}$$

The external moment of the 10,000 lb. force about C is

$$M = 10{,}000 \times 5 = 50{,}000 \text{ in.-lb.}$$

and the resisting moment must equal the external moment

$$\begin{aligned} 50{,}000 &= 4.408S_2 \\ S_2 &= 11{,}340 \text{ lb. per sq. in. on } A \text{ or } D \end{aligned}$$

For rivet A, $S_1 = 5660$ lb. per sq. in. to the left and $S_2 = 11{,}340$ lb. per sq. in. upward. Their resultant will be

$$\begin{aligned} S_A &= \sqrt{5660^2 + 11{,}340^2} \\ &= 12{,}690 \text{ lb. per sq. in.} \end{aligned}$$

For rivet E, $S_1 = 5660$ lb. per sq. in. to the left and $S_2 = 11{,}340$ lb. per sq. in. downward. Their resultant will be

$$\begin{aligned} S_B &= \sqrt{5660^2 + 11{,}340^2} \\ &= 12{,}690 \text{ lb. per sq. in.} \end{aligned}$$

Problems

100. An 8-in. 23-lb. per ft. I-beam is used to support two 4000-lb. loads as shown in Fig. 125. A compressive force P of 12,420 lb. is applied horizontally at the center of gravity of the cross section. Neglecting the weight of the beam, what unit stress results at A and B of the cross-section 6 ft. from either reaction?

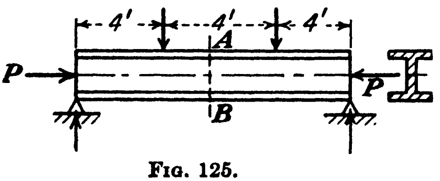


FIG. 125.

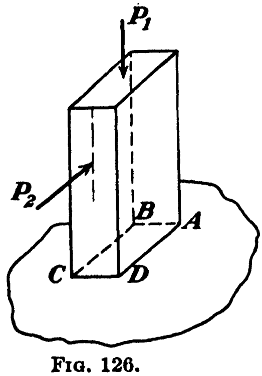


FIG. 126.

101. Figure 126, $AB = CD = 3$ in., $AD = BC = 6$ in. shows a short post fixed to the floor. A load P_1 of 3600 lb. is applied to the center of gravity of the cross section and $P_2 = 1800$ lb. is applied horizontally 8 in. above the base of the post. What is the unit stress along edges AB and CD?

102. A cantilever *T* beam has $I_{1\text{-}1} = 5.7$ in.^{4} What unit stress results at *A* and at *B* (Fig. 127) when $P = 5000$ lb. is applied to the center of gravity of the cross section? The area of the *T* is 4 sq. in.

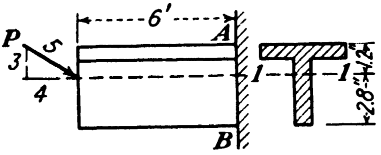


FIG. 127.

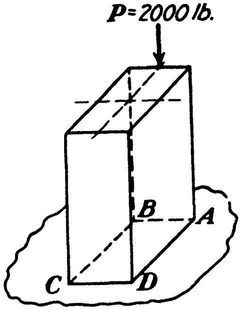


FIG. 128.

103. A short timber post, Fig. 128, has a maximum allowable stress of 1000 lb. per sq. in. The side $AB = CD = b$ and $BC = AD = h$. When $b = \frac{1}{2}h$ what size timber is required?

104. In Fig. 129, calculate the unit stress at *A* and *B* when $P = 3000$ lb.

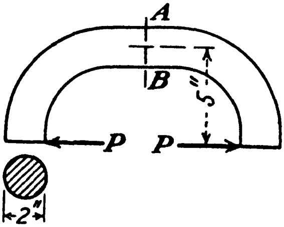


FIG. 129.

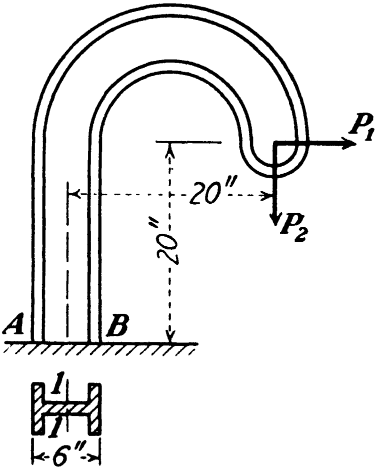


FIG. 130.

105. A machine part, Fig. 130, is acted upon by P_1 and P_2. The maximum unit stress at *A* or *B* is not to exceed 20,000 lb. per sq. in. When $P_2 = 5000$ lb., what force P_1 may be applied? The cross section is that of a 6-in. I-beam at 17.25 lb. per ft.

106. An eccentrically loaded riveted connection is shown in Fig. 131. What unit shearing stresses result in rivets *A*, *B*, and *C*?

107. A riveted connection using $\frac{7}{8}$-in.-diameter rivets is shown in Fig. 132. If the connection has two rivets *A* and *B*, what unit shearing stress results in each of the rivets? If a third rivet were driven at D, what shearing stress would result in each of rivets A, B, and D? What is the ratio of the maximum shearing stress in A or B when two rivets are used to the maximum shearing stress in A, B, or D when three rivets are used? Which connection (two or three rivets) would you consider safer?

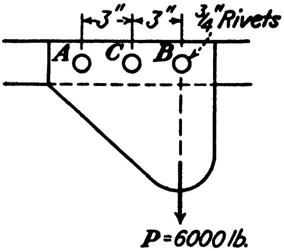


FIG. 131.

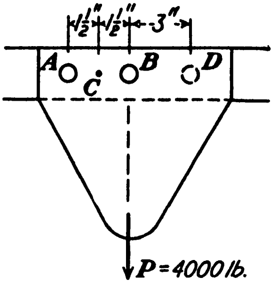


FIG. 132.

108. In the eccentrically loaded riveted connection shown in Fig. 133, four fundamental shearing stresses are set up in each rivet. By dividing the load into horizontal and vertical components, each component causes a direct and rotational shearing stress in each rivet. What is the resulting shearing stress in each rivet?

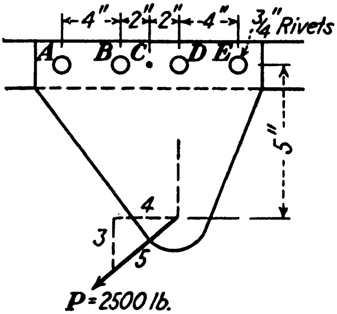


FIG. 133.

CHAPTER X

COLUMNS

When a piece of material, which is long compared with the least cross section dimension, is acted upon by compressive forces, it will fail by bending sideways as shown in Fig. 136. This type of failure is called "buckling" and the piece is said to be a **column.** If Figs. 134, 135, and 136 are observed, the piece of timber in each case having the same cross section area, but with a length L longer than the preceding one, it will be seen that the timber in Figs. 135 and 136 is greatly deformed and that buckling is taking place. This failure is a combination of compression, shear and bending.

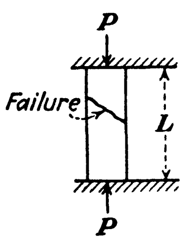


FIG. 134.

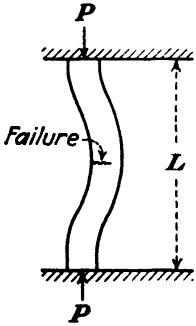


FIG. 135.

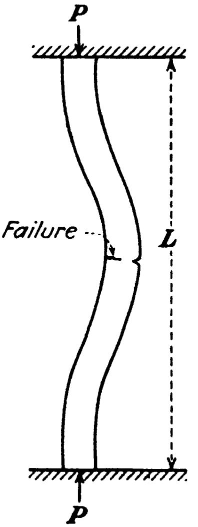


FIG. 136.

Since a column fails due to a combination of stresses, the design or selection of a size which will be safe *cannot* be made by the direct compression equation,

$$P = SA$$

but must be made by a formula which will take into account the combination of stresses.

The method, used in practice at the present time, is to assume a certain safe direct unit compressive stress and reduce it in proportion to the length of the column, the **radius of gyration,** and the end conditions.

The American Institute of Steel Construction recommends that the following formula, which has been widely adopted, be used for the design of steel columns:

$$\frac{P}{A} = 17{,}000 - 0.485\frac{L^2}{r^2}$$

where P = total (axial) load to be supported (pounds)
A = cross section area (square inches)
L = length of the column (inches)
r = least radius of gyration (inches)

The least radius of gyration is equivalent to the square root of the least moment of inertia divided by the cross section area or

$$r = \sqrt{\frac{I}{A}}$$

In recent years designers and steel companies have developed a large series of *H* steel sections, similar to *I*-beams, but with the flange width and depth equal or nearly so. These are listed in the Appendix under the heading CB sections, along with the sections which are used entirely as beams. The sizes used as columns are:

CB 14 × 16
CB 14 × 14½
CB 14 × 10
CB 14 × 8
CB 12 × 12
CB 12 × 10
CB 12 × 8
CB 10 × 10
CB 10 × 8
CB 8 × 8
CB 8 × 6½

There is no reason why any of the other sections in the Appendix, but not tabulated above, could not be used as columns except for the difficulty in fabrication and lack of economy.

For any section, using the 14 in. × 16 in. at 246 lb. per ft. *CB* section as an example, there are two radii of gyration listed about the 1-1 axis and about the 2-2 axis. The columns will fail in such a way that the bending or buckling will have the 2-2 axis as the neutral axis, and as the radius of gyration for this axis is the *least* of the radii of gyration, it will *always* be used.

The term L/r is called the **slenderness ratio** of a column and is limited in practice to a value not to exceed 120. In some cases where the column is of minor importance in the structure, values of L/r may exceed 120 although good practice recommends that the limit of 120 not be exceeded.

The selection of a structural steel column is made by a method of trial as:

Example.—What size column is necessary to support an axial load of 150,000 lb. if the length of the column is 10 ft.?

This information is tabulated in the A.I.S.C. Handbook. The principle of determining the proper size is a method of trial, but certain factors reduce the number of available sizes to a few.

From the equation

$$\frac{P}{A} = 17{,}000 - 0.485\frac{L^2}{r^2}$$

it will be seen that the value of P/A must be less than 17,000 and will approximate 15,000. Then when $P = 150{,}000$ lb. and P/A is approximately 15,000, A will equal $\frac{P}{15{,}000}$, or

$$A = \frac{150{,}000}{15{,}000} = 10 \text{ sq. in. (approximate area required)}$$

Looking in the CB Sections list, try the smallest column which has the approximate area. This is an 8 × 8 in. weighing 35 lb. per ft. with an area of 10.30 sq. in. For this size and weight, the radius of gyration r about the 2-2 axis is given as 2.03. (It should be noted that the values of r about the 2-2 axis for all of the 8 in. × 8 in. weights do not vary greatly and hence the term $0.485\frac{L^2}{r^2}$ will only change slightly for these weights.) Then substituting in the equation:

$$\frac{P}{A} = 17{,}000 - 0.485\frac{L^2}{r^2}$$

$$P = A\left(17{,}000 - 0.485\frac{L^2}{r^2}\right)$$

$$\begin{aligned} P &= 10.30\left(17{,}000 - 0.485\frac{(120)^2}{(2.03)^2}\right) \\ &= 10.30(17{,}000 - 1690) \\ &= 10.30(15{,}310) \\ &= 157{,}000 \text{ lb.} \end{aligned}$$

This size H section is safe as the load to be supported is 150,000 lb. while it will safely support 157,000 lb. Also, $\frac{L}{r} = \frac{120}{2.03} = 59.1$ which is less than the specified maximum.

A trial should be made to see if the next lighter section which is 8 in. × 8 in. weighing 33 lb. per ft. will be satisfactory.

Then

$$
\begin{aligned}
A &= 9.70 \text{ sq. in.}; \quad r = 2.02 \\
P &= A\left(17{,}000 - 0.485\frac{L^2}{r^2}\right) \\
&= 9.70\left(17{,}000 - 0.485\frac{(120)^2}{(2.02)^2}\right) \\
&= 9.70(17{,}000 - 1710) \\
&= 9.70(15{,}290) \\
&= 148{,}200 \text{ lb. which is less than the load to be supported, and hence is not safe.}
\end{aligned}
$$

We know now that the 8 in. × 8 in. at 35 lb. per ft. *H* column is safe and should be used. A larger size would be uneconomical due to the increased weight.

The same formula, $\frac{P}{A} = 17{,}000 - 0.485\frac{L^2}{r^2}$ is recommended for the design of steel columns having rectangular or circular cross sections. The equations for the radius of gyration for various shaped areas are listed in the Appendix under the title Elements of Sections.

Referring to this section, it will be seen that the radius of gyration for a solid rectangle about the 1-1 axis is given, as

$$r_{1\text{-}1} = \frac{d}{\sqrt{12}}$$

where d = larger dimension in inches. The 2-2 axis (of the *least* radius of gyration) is not shown but the radius of gyration about this axis is

$$r_{2\text{-}2} = \frac{b}{\sqrt{12}}$$

where b = least dimension in inches.

For a solid circular cross section,

$$r_{1\text{-}1} = r_{2\text{-}2} = \frac{d}{4}$$

where d = diameter in inches.

For a hollow circular cross section,

$$r_{1\text{-}1} = r_{2\text{-}2} = \frac{\sqrt{d^2 + d_1^2}}{4}$$

where d = external diameter and d_1 = internal diameter.

Example.—What axial load will a steel column $1\frac{1}{2}$ in. × 2 in. × 36 in. long safely support?

$$r_{2\text{-}2} = \frac{b}{\sqrt{12}} = \frac{1.5}{\sqrt{12}} = 0.433$$

To check L/r:

$$\frac{L}{r} = \frac{36}{0.433} = 83 \text{ which is less than } 120$$

Then

$$\frac{P}{A} = 17{,}000 - 0.485\frac{L^2}{r^2}$$

$$P = A\left(17{,}000 - 0.485\frac{L^2}{r^2}\right)$$

which is equivalent to writing the formula

$$\begin{aligned} P &= A\left[17{,}000 - 0.485\left(\frac{L}{r}\right)^2\right] \\ &= 2 \times 1.5[17{,}000 - 0.485(83)^2] \\ &= 3(17{,}000 - 3340) \\ &= 3 \times 13{,}660 \\ &= 40{,}980 \text{ lb. safe load.} \end{aligned}$$

Example.—A hollow pipe column is 10 ft. long. The external diameter is 4.50 in. and the internal diameter is 4.026 in. (These are the diameters of 4-in. standard pipe.) What safe concentric (axial) load will the column support?

$$r_{2\text{-}2} = \frac{\sqrt{d^2 + d_1^2}}{4} = \frac{\sqrt{(4.50)^2 + (4.026)^2}}{4} = \frac{\sqrt{20.25 + 16.21}}{4}$$

$$= \frac{\sqrt{36.46}}{4} = \frac{6.04}{4}$$

$$r_{2\text{-}2} = 1.51 \text{ in.}$$

$$A = \frac{\pi}{4}d^2 - \frac{\pi}{4}d_1^2 = \frac{\pi}{4}(d^2 - d_1^2) = \frac{\pi}{4}(20.25 - 16.21)$$

$$= \frac{\pi}{4}(4.04) = 3.17 \text{ sq. in.}$$

To check $\frac{L}{r}$:

$$\frac{L}{r} = \frac{120 \text{ in.}}{1.51 \text{ in.}} = 79.5 \text{ which is less than } 120$$

Then:

$$\begin{aligned} P &= A\left(17{,}000 - 0.485\frac{L^2}{r^2}\right) \\ &= A\left[17{,}000 - 0.485\left(\frac{L}{r}\right)^2\right] \\ &= 3.17[17{,}000 - 0.485(79.5)^2] \\ &= 3.17(17{,}000 - 3060) \\ &= 3.17 \times 13{,}940 \\ &= 44{,}100 \text{ lb. safe load.} \end{aligned}$$

In the design of machine parts, very often other equations are specified and must be used. Such a design entails no additional difficulties as the same general procedure is followed except that a value of L/r greater than 120 may be permitted.

Problems

All problems are to be solved by using the American Institute of Steel Construction formula. The value of L/r must not exceed 120.

109. A steel column is 1 in. × ½ in. × 16 in. long. What load will the column support?

110. A piston rod, 2 in. in diameter, is 36 in. long. What load will it support?

111. If the piston rod in problem 110 is attached to a piston 15 in. in diameter, what steam pressure is safely admissible to the cylinder?

112. A 5-in. extra strong steel pipe is to be used as a column 16 ft. long. The external diameter is 5.563 in. and the internal diameter is 4.813 in. The net area of the pipe is 6.11 sq. in. What is the radius of gyration of the pipe?

113. What load will the pipe column of problem 112 support?

114. A CB 10 in. × 8 in. column weighing 41 lb. per ft. has the following data given for its cross section:

$$\begin{aligned} A &= 12.06 \text{ sq. in.} \\ r_{1\text{-}1} &= 4.29 \text{ in.} \\ r_{2\text{-}2} &= 1.99 \text{ in.} \end{aligned}$$

Is the column safe to support a load of 170,000 lb. if it is 12 ft. long?

115. A 3-in. American Standard I-beam is used as a column 5⅙ ft. long. What safe load will it support when the following data applies to its cross section:

$$\begin{aligned} A &= 1.64 \text{ sq. in.} \\ r_{1\text{-}1} &= 1.23 \text{ in.} \\ r_{2\text{-}2} &= 0.53 \text{ in.} \end{aligned}$$

Wood Columns.—The design and selection of a timber column requires a slightly different procedure than does the design of a steel column. The most recent and thorough study of timber columns has been made by the Forest Products Laboratory of the U. S. Forest Service. The results of this study have been adopted by the American Society of Testing Materials and are recommended as good practice for general use.

Timber columns are classified according to the ratio L/d where L is the length in inches and d is the least dimension of the cross section in inches.

Class 1. When the value of L/d does not exceed 10, the timber post (column) shall be designed using the compression equation

$$S = \frac{P}{A}$$

where S = safe working stress of the timber in pounds per square inch (see Table 6)
P = total load in pounds
A = cross-section area in square inches

TABLE 6.—SAFE WORKING STRESSES, POUNDS PER SQUARE INCH
Common Structural Grade, Continuously Dry

Species	Values of L/d											
	10 or less	12	14	16	18	20	25	30	35	40	50	E*
Douglas fir												
Coast region........	880	870	861	847	826	796	675	487	358	274	175	1.6
Dense..............	1025	1017	996	965	935	893	698	487	358	274	175	1.6
Hemlock												
West Coast.........	720	712	706	696	680	660	573	426	313	240	153	1.4
Larch												
Western............	880	863	849	828	798	752	570	396	291	223	142	1.3
Pine												
Southern...........	880	870	861	847	826	796	675	487	358	274	175	1.6
Dense..............	1025	1017	996	965	935	893	698	487	358	274	175	1.6
Spruce												
White..............	640	632	627	617	602	582	500	365	268	206	132	1.2
Oak												
Red or white........	800	791	783	771	753	729	626	457	336	257	164	1.5

* Values of E are to be multiplied by 1,000,000. Thus for West Coast hemlock. $E = 1.4 \times 1{,}000{,}000 = 1{,}400{,}000$.

The student is referred to a handbook for similar tables of Safe Working Stresses for timber columns used where the conditions are occasionally wet or continuously wet. Also for the tables for the Select Grade Timber.

Example of Class 1 *Timber Column.*—A common grade pine timber post is 3 ft. long and 4 in. × 4 in. in cross section. What safe load can it support?

$$\frac{L}{d} = \frac{36 \text{ in.}}{4 \text{ in.}} = 9 \qquad \text{(therefore this is a class 1 column)}$$

From Table 6, opposite Southern pine, find 880 lb. per sq. in. in the column of figures for L/d of 10 or less.

Then

$$S = \frac{P}{A} \qquad \text{and} \qquad P = SA$$

Hence

$$P = 880 \times 4 \times 4 = 14{,}080 \text{ lb.},$$

which is the safe working load that the post will support if the common grade Southern pine is continuously dry.

Class 2. When the value of L/d is greater than 10 but less than 25 (intermediate length columns), the following formula shall apply

$$\frac{P}{A} = S\left[1 - \frac{1}{3}\left(\frac{L}{kd}\right)^4\right] \qquad \text{eq. } (A)$$

where P = total load in pounds
A = cross-section area in square inch
S = safe working stress for short (L/d less than 10) columns (see note following)
L = length in inches
d = least dimension of cross-section in inches
k = a constant depending on the grade and kind of timber and the conditions under which the column is to be used (see Table 7)

The formula above is called the Forest Products Laboratory Formula for intermediate length columns; this is abbreviated F.P.L. intermediate column formula.

Note: The meaning of S will appear inconsistant with the class of columns to which this formula applies, but S correctly represents the safe working stress for a short column. Inasmuch as the numerical result of the term within the brackets is always less than one, this result is the factor by means of which the safe working stress for a short column is reduced so that it correctly applies to an intermediate length column. These facts are illustrated in the following numerical example by two solutions for the same given data.

TABLE 7.—VALUES OF K FOR TIMBER COLUMNS OF INTERMEDIATE LENGTH

Common Structural Grade, Continuosly Dry

Species	k
Douglas fir	
Coast region	27.3
Dense	25.3
Hemlock	
West Coast	28.3
Larch	
Western	24.6
Pine	
Southern	27.3
Dense	25.3
Spruce	
White	27.8
Oak	
Red or white	27.8

The student is referred to a handbook for values of k for other grades of all species and for other conditions of use. However, the values of safe working stresses as given in Table 6 will eliminate the major computations required when using the F.P.L. intermediate column formula. Thus the formula becomes

$$\frac{P}{A} = S \qquad \text{(eq. } B\text{)}$$

where S is the safe working stress from Table 6 for the proper species and value of L/d.

Example of Class 2 Timber Column.—A common grade Southern pine column is 6 in. × 8 in. × 10 ft. long. What load will it safely support?

$$\frac{L}{d} = \frac{10 \times 12}{6} = \frac{120}{6} = 20 \text{ (therefore this is a class 2 column)}$$

Using eq. (B):

$$\frac{P}{A} = S$$

From Table 6, opposite Southern pine, find 796 lb. per sq. in. in the column of figures for $L/d = 20$. Then

$$P = AS = 6 \times 8 \times 796 = 38{,}208 \text{ lb. (safe load)}$$

To check, and show the use of the complete F.P.L. formula for intermediate columns:

$$\frac{P}{A} = S\left[1 - \frac{1}{3}\left(\frac{L}{kd}\right)^4\right]$$

From Table 6

$$S = 880 \text{ for short columns}$$

From Table 7

$$k = 27.3$$
$$A = 6 \times 8 = 48 \text{ sq. in}$$
$$\frac{L}{d} = 20$$

Therefore

$$\begin{aligned} P &= AS\left[1 - \frac{1}{3}\left(\frac{L}{kd}\right)^4\right] \\ &= 48 \times 880\left[1 - \frac{1}{3}\left(\frac{20}{27.3}\right)^4\right] \\ &= 48 \times 880\,[1 - \tfrac{1}{3}(0.288)] \\ &= 42{,}240[1 - 0.096] \\ &= 42{,}240 \times 0.904 = 38{,}185 \text{ lb. (which checks closely enough)} \end{aligned}$$

It is necessary to interpolate between the values of L/d given in Table 6 for the correct values of S.

Example.—What load can a common grade Southern pine column 6 in. × 8 in. × 11 ft. long safely support?

$$\frac{L}{d} = \frac{11 \times 12}{6} = 22$$

In Table 6, for Southern pine, when

$$\frac{L}{d} = 20, \qquad S = 796$$

and when

$$\frac{L}{d} = 25, \qquad S = 675$$

It will be seen that 22 is $\frac{2}{5}$ of the way between 20 and 25, therefore $\frac{2}{5}$ of the difference in the values of $S = 796$ and $S = 675$ is $\frac{2}{5} \times 121 = \frac{242}{5} = 48.4$ which is to be subtracted from the value of $S = 796$ when $L/d = 20$.
Hence

$$S = 796 - 48.4 = 747.6 \qquad \text{when} \qquad \frac{L}{d} = 22$$

Then, using eq. (*B*)

$$\begin{aligned} \frac{P}{A} &= S \\ P &= AS \\ &= 6 \times 8 \times 747.6 \\ &= 35{,}885 \text{ lb. (safe load)} \end{aligned}$$

Class 3. When the value of L/d is 25 or greater, but less than 50 (long columns) the following formula shall apply:

$$\frac{P}{A} = \frac{E\pi^2}{36\left(\frac{L}{d}\right)^2} \qquad \text{eq. } (C)$$

where P = total load in pounds
A = cross-section area in square inches
E = modulus of elasticity (Table 6)
L = length in inches
d = least dimension in inches

This formula is called the Forest Products Laboratory Formula for long columns. It is abbreviated F.P.L. long column formula.

Note: **Timber columns shall be limited to a maximum value of L/d = 50.** The student is referred to a handbook for values of E for other species of timber and for other conditions of use. However, Table 6 will again eliminate the computations necessary with this (F.P.L. long column) formula by giving correct safe working stresses when L/d exceeds 25. Thus eq. (*C*) becomes

$$\frac{P}{A} = S \qquad \text{eq. } (B)$$

where S is the safe working stress from Table 6 for the proper species and value of L/d.

Example of Class 3 Timber Column.—A common grade Southern pine column is 6 in. × 8 in. × 15 ft. long. What load will it safely support?

$$\frac{L}{d} = \frac{15 \times 12}{6} = 30 \text{ (therefore this is a class 3 column)}$$

Using eq. *B*

$$\frac{P}{A} = S$$

From Table 6, opposite Southern pine find 487 lb. per sq. in. in the column of figures for $L/d = 30$.

Then

$$P = AS = 6 \times 8 \times 487 = 23{,}380 \text{ lb. (safe load)}$$

To check, and show the use of the complete F.P.L. long column formula:

$$\frac{P}{A} = \frac{E\pi^2}{36\left(\frac{L}{d}\right)^2}$$

From Table 6, for Southern pine, $E = 1{,}600{,}000$ lb. per sq. in.

$$A = 48 \text{ sq. in.}$$

$$\frac{L}{d} = 30$$

Therefore

$$P = \frac{A\pi^2 E}{36\left(\frac{L}{d}\right)^2} = \frac{48\pi^2 \times 1{,}600{,}000}{36 \times (30)^2}$$

$$= 23{,}380 \text{ lb. (safe load)}$$

To determine the correct value of S, for columns whose L/d is not given in Table 6, it is necessary to interpolate.

Problems

The following columns are to be made of continuously dry, common grade timber; the safe load will be calculated by the Forest Products Laboratory recommendations; in all problems, state the class (1, 2, or 3) to which the column belongs.

116. What safe load will a white spruce column 2 in. × 2 in. × 3 ft. safely support?

117. A West Coast hemlock post is 6 ft. long and 4 in. × 6 in. in cross section. What safe load can it support?

118. A piece of dense Douglas fir 6 in. × 8 in. × 1 ft. long is used to support a load of 24 tons. Is it safe?

119. Is a dense pine post 10 ft. long and 4 in. × 4 in. safe if it supports a load of 8,000 lb.?

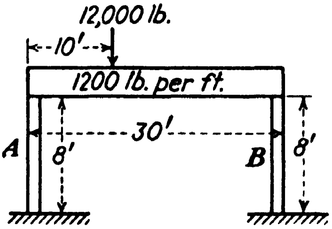


Fig. 137.

120. Post *A* is used to support the left end of the beam shown in Fig. 137. Will a 6 in. × 8 in. post be safe if made of hemlock?

121. Will a 6 in. × 6 in. hemlock post *B* be safe to support the right end of the beam shown in Fig. 137?

CHAPTER XI

MATERIALS

In order to be able to make a selection of a material to safely resist the forces which will be imposed upon it and the stresses set up within it, an understanding of the properties of materials in general is necessary.

The mechanical properties of a material refer to the characteristics of the material and its behavior under stress. These characteristics are indicated by the following terms which are defined.

A material is said to be **ductile** when it can be drawn out or stretched without breaking. Wrought iron, soft steel, and copper are several of the ductile metals. Ductility is measured by the per cent elongation—that is, the ratio in per cent of the stretch to the original length.

A material is said to be **brittle** when it can be deformed a very small amount before it breaks. Generally there is no warning before it fails suddenly. Cast iron, brick, hard steel are examples of brittle materials.

Flexible materials are those which will bend considerably without rupture. Spring steel is an example of a flexible metal, while hickory is a flexible wood.

A **tough** material is one which is capable of absorbing heavy shocks. Wrought iron and mild steel are said to be tough. Manganese steel is also tough.

A material is said to be **malleable** when it can be hammered out into thin sheets without rupture. Copper is very malleable while wrought iron and soft steel are only somewhat malleable.

Hard materials are those which offer great resistance to *permanent* deformation. That is, they resist being drilled, scratched, etc. White cast iron is a hard metal.

By knowing the mechanical properties of the various materials, the selection of a material to satisfactorily and safely perform a certain job is narrowed down to a few materials. From these few materials, the selection is then dependent upon the economic factors in which the cost of the finished product is compared with the cost of fabrication, machining, heat treatment, ease of reproducibility, and spoilage.

Wrought iron is a material composed of two principal constituents: highly refined metallic iron and slag. The two constituents, in the ratio of about 97% iron to 3% slag, are finely intermixed although no complete union (fusion) takes place between them. Due to the squeezing and rolling of the material during the manufacturing process, the iron and slag are firmly rolled together so that the slag is in the form of very small broken threads parallel to the direction of rolling. This causes the wrought iron to be known as a directional material that is, its strength varies with the direction of rolling. When a tensile force P_2 is applied to the piece of wrought iron plate perpendicular to the direction of rolling, as shown in Fig. 138, a tensile unit stress (S_2) on the cross section area A_2 is set up.

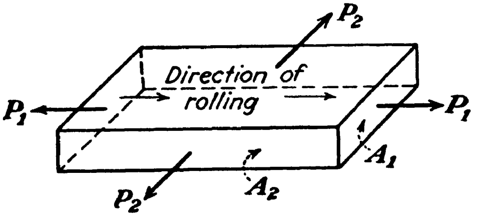


FIG. 138.

If a tensile force P_1 is applied in the direction of rolling to a similar piece of the *same* wrought iron plate, a tensile unit stress (S_1) on the cross section area A_1 is set up. By extensive tests, the ratio of S_2 to S_1 has been found to average about 75%. Thus, it is possible to state that the unit tensile stress perpendicular to the direction of rolling is about 75% of the unit tensile stress in the direction of rolling.

A greater cohesion between the slag and the iron, and a finer grained material results from more rolling. Hence the tensile strength of wrought iron is greater for smaller bars than for larger bars.

A very valuable property of wrought iron is the ease with which it can be welded or hammered together while at a high heat.

There are two defects in wrought iron which may take place when the iron is being rolled or forged. The first is called **red-**

short wrought iron if the iron crumbles while being worked at a red heat. This is due, principally, to the use of a slag with a high sulphur content.

The second is called **cold-short** wrought iron if the iron crumbles while being worked cold. This is due to the use of a slag with a high phosphorous content.

A comparatively new development in the wrought iron field is the alloy—nickel wrought iron. The addition of about 3% of nickel to wrought iron increases the ultimate strength 25% and materially increases the resistance to corrosion. The increased strength is in some measure offset by the decreased ductility.

Steel is a material which has its mechanical properties affected by:

1. The process of manufacture
2. The chemical composition
3. The mechanical work (rolling, etc.) done on it
4. The heat treatment

It is defined as a material which is composed principally of iron and carbon, can be hardened by sudden cooling, and is sufficiently malleable to be rolled into shapes. In addition to the iron and carbon, small amounts of silicon, phosphorous, sulphur and manganese are present as well as minute quantities of other metals such as copper.

Carbon exerts a very pronounced influence on the strength, ductility and hardness of the steel, resulting in the following trends:

As the amount of carbon is gradually increased from a negligible quantity to 0.85%, the tensile and compressive strengths will be increased from 40,000 lb. per sq. in. to a maximum of 120,000 lb. per sq. in. A further increase in the amount of carbon to about 1.7% will result in a decrease in the tensile strength to about 104,000 lb. per sq. inch.

As the amount of carbon is gradually increased from a negligible amount to 1.7%, the ductility is gradually decreased while the hardness is increased. Thus, it will be noticed that the hardness of steel increases with every addition of carbon, but the tensile strength increases as some carbon is added and then decreases as more is added.

Like the ductility of steel, the toughness decreases as the carbon content increases.

Mechanical working of steel can be divided into two classes:

1. Hot work (work which is performed at a temperature of 1350°F. or more).

2. Cold work (work which is performed at a temperature well below 1350°F.).

Hot work such as rolling and forging is very beneficial to steel, resulting in increased strength, increased ductility and decreased hardness. The increased strength is due primarily to a refinement of the granular structure of the steel.

Cold work such as cold rolling or wire drawing produces an increase in tensile strength by permanently deforming the steel. However, the effect on the hardness is to greatly increase it, at the same time lowering the ductility.

Heat treatment of steel is the heating and cooling of steel for the purpose of producing certain desirable conditions and properties. It should be understood that heating for the purpose of mechanical working is excluded from the purpose of the heat treatment.

Annealing is an inclusive term and should be used with caution. It is defined as a heating and cooling operation usually with a relatively slow cooling. The principal object of all annealing operations is to soften the steel or to decrease its hardness.

The two annealing operations which have the greatest use are normalizing and tempering.

Normalizing is the heating to a temperature usually from 1450°F. to 1700°F., depending on the carbon content, followed by cooling in still air. These temperatures will vary with the kind of steel being normalized, the values given being for plain carbon steels. This is the process which is ordinarily called annealing, but as previously explained, all heat treatments which are used to decrease the hardness and increase the ductility are called annealing operations.

While the normalizing treatment is used to decrease the hardness, it will materially increase the ductility. Its effect upon the strength of the steel is to decrease it as shown in the following group of values:

Tempering operations will be defined and explained following the hardening process.

Hardening is the heating and quenching from a temperature above 1350°F. As the term implies, this heat-treating operation

TABLE 8

Tensile strength, pounds per square inch			% elongation	
% carbon	As hot rolled	Then normalized	As hot rolled	Then normalized
0.14	60,000	55,000	38	40
0.32	73,000	70,000	30	35
0.49	96,000	86,000	24	27
0.83	140,000	115,000	12	16
1.01	153,000	108,000	8	22
1.39	148,000	100,000	1	19

is used to increase the hardness of the steel, considering at the same time, the effect on the other mechanical properties.

There are two essential factors to be kept in mind when hardening a carbon steel: (1) The speed at which the piece of steel can be cooled to a temperature of less than 1350°F., and (2) the percent of carbon present.

It is well known that steel, upon being heated, undergoes a complete change in its internal structure when the temperature increases from about 1350°F. to from 1450°F. to 1700°F. Upon cooling through approximately the same temperature range, the new structure reverts to the original structure except as to the size of the grains which make up the piece.

1. In reverting to the original structure, the hardness property of the steel is affected in a manner, dependent upon the speed with which the pieces can be cooled (the amount of carbon being constant):

		Hardness	Other Properties
Cooled from 1600°F.	very rapid	Fair	
	rapid	Extremely hard	Very brittle and non-malleable.
	fairly rapid	Hard	
	slow	Moderately hard	Excellent strength and toughness.
	very slow	Soft	Very ductile.

If the piece of steel is large or thick, often times the external portion, after quenching or cooling, is harder than the internal portion of the piece because the heat cannot be reduced at the same rate. Thus the exterior will be harder than the interior.

2. From the results of many experiments, the conclusion was reached that steel with a carbon content of about 0.85% can be hardened to the greatest degree (Table 9) for plain carbon steels. However, steels with a carbon content of from 0.4% to 1.2% seem to possess the property of being increased in hardness the same amount. That is, the difference (Table 9*) in their hardness before and after hardening is about the same. The hardness of metals is measured by several methods, one of which will be explained. The **Brinell** method of measuring hardness is to push a 10-mm.-diameter steel ball into the metal by a force of 3000 kg. (sometimes less force is used depending on the kind and thickness of the piece) and then measuring the diameter of the indentation in the surface of the piece. By means of a mathematical equation, the unit pressure on the inside surface of the indentation is calculated, the unit pressure being called the Brinell hardness.

As an additional explanation of the effect of the carbon content on the hardness, the following Brinell hardness values are given:

TABLE 9.—BRINELL HARDNESS NUMBERS

Carbon content	As annealed	As hardened	Difference
0.1	100	150	50
0.2	120	200	80
0.4	170	470	300
0.6	240	640	400*
0.8	240	650† (maximum)	410*
0.9	250	630	380*
1.0	250	630	380*
1.2	260	630	370*

* The values given are approximate but serve to show the maximum hardness (†) and the fairly constant differences (‡) for steels of from 0.4% to 1.2% carbon. All pieces tested were quenched at a very rapid rate.

If a hardened steel is found to be too hard, a tempering (drawing) process is used to decrease the hardness. **Tempering** is the reheating *after hardening* to some temperature less than 1250°F. followed by any rate of cooling.

The tempering process can be explained as a process to relieve some of the hardness induced by the quenching operation, the amount of the hardness being relieved depending on the temperature to which the hardened steel is reheated. The higher the temperature to which the steel is reheated, the greater the part of the hardness relieved. Thus, although there should be no reason for doing it, a hardened piece of steel can be restored to its original hardness by reheating to about 1350°F. A 0.3% carbon steel, as quenched, shows a Brinell hardness of about 420.

If reheated to 200°F. the hardness is reduced to 390.
If reheated to 400°F. the hardness is reduced to 330.
If reheated to 600°F. the hardness is reduced to 270.
If reheated to 800°F. the hardness is reduced to 240.
If reheated to 1000°F. the hardness is reduced to 220.
If reheated to 1200°F. the hardness is reduced to 200.
If reheated to 1400°F. the hardness is reduced to 150.

The trend of the tempering process is well illustrated in the above group of Brinell hardness numbers, and it follows directly that similar steels with different percentages of carbon are affected in a similar manner. It must be constantly kept in mind that the other mechanical properties are being altered to a more or less extent although the hardness property of the steel is the principal property under consideration.

Case hardening, as the term implies, is the process of making the exterior of a piece of steel hard while the interior remains fairly tough and ductile. The principle of this process is the ease with which the surface of red-hot low carbon steel and iron parts absorb carbon. After the surface has absorbed sufficient carbon from the surrounding carbon packing, the piece is quenched. Thus the surface is made highly resistant to wear while the interior is still capable of absorbing shocks or is very tough.

STEEL ALLOYS

Very often, it is desirable to have a material with certain mechanical properties which are not available in the plain carbon steels. By adding a small amount of a certain other metal or metals, an alloy steel with the desired properties can often be made.

Generally, the alloys can be heat-treated much more easily than a plain steel, and this factor alone has increased the demand for some alloy steels.

As the number of alloy steels available on the market at the present time is large, the discussion will be confined to a few of the more widely used alloys. Alloy steels cost more than plain carbon steels, especially those alloys steels for which there is little demand.

Nickel steel is made in two grades, low nickel steel where the nickel varies from 3.0% to 4.5%, and high nickel steel where the per cent of nickel varies from 30 to 36. These steels, considered as one class, are made in larger amounts and have more usages than any other alloy steel.

The low nickel group contains the structural nickel steel which is made in large quantities. Steels of this group show an increased tensile strength and hardness, although when a plain carbon steel and a nickel steel with the *same* tensile strength are compared, the nickel steel will show a slightly greater per cent elongation (ductility). These steels are somewhat better than carbon steels in resisting corrosion.

In the high nickel steel group, the materials are very strong and tough and are highly resistant to corrosion. Included in this group is **invar,** (36% nickel), a material which expands only 8.6% as much as plain steel. Its expansion is practically negligible which makes the use of invar valuable in watches and measuring tapes.

Chromium steel, otherwise called **chrome steel** provides a material which has a high elastic limit and a great degree of hardness. Its use is in parts where wear is a primary factor, such as in bearings, projectiles, safes, etc. **Stainless steel,** which is a chrome steel, is used principally for cutlery and surgical instruments and contains about 14% chromium. It is exceptionally resistant to corrosion.

Chrome-nickel steels have high strength and hardness coupled with good ductility and toughness. They are used for armor plate and to a large extent in the automobile industry for gears and shafts.

Chrome-vanadium steels can be machined easily and can be heat treated. They have excellent strength and hardness and are very resistant to breakage under the action of repeated stresses (fatigue). Thus they are used for automobile springs, dies, and bearing race ways.

Nickel-manganese steels are exceptionally tough and must be cast into their final shape and finished by grinding. Their use is in applications requiring great toughness and strength such as for crusher jaws and railroad frogs.

High speed steels are the tool steels containing a number of different elements. Examples are: iron, carbon, tungsten, chromium, vanadium, and cobalt as well as small amounts of sulphur, silicon, and phosphorous. These steels are capable of keeping a cutting edge when run at a red heat, and, in fact, some work to their best advantage only when run at a red heat.

CAST IRON

Cast iron is a material much like steel but contains so much carbon that it is not usefully malleable at any temperature. The carbon content varies from 1.7% to about 4.5% but unlike the carbon in steel, it may appear in the cast iron in either or both of two distinct forms: (1) as free carbon in the form of flakes, or (2) chemically combined with the iron.

The form of the carbon influences the appearance of a broken section of the iron, the free carbon producing a grayish color and the combined carbon producing a silvery white color. If both forms are present with the combined carbon predominating, the color will be a combination of gray and white coloring and is called mottled cast iron.

Cast iron is a material which is inherently weak in tension and shear but is strong in compression. It is hard and brittle and possesses little ductility. Hence the discussion of cast iron will be confined to the compressive strength and hardness.

A very pronounced effect on the compressive strength of cast iron is obtained by varying either the total amount or the form of the carbon present. If carbon in increasing amounts is present in cast iron, and all of the carbon is in the combined form, the compressive strength increases as the per cent of carbon increases while the ductility is practically zero. The hardness likewise increases from a Brinell hardness of 260 at 1.7% total carbon (combined) to a Brinell hardness of 450 at 4.5% total carbon (combined).

When a cast iron of 4% carbon is considered, as an example, the form of the carbon varying from being entirely combined to entirely free, the compressive strength decreases from 280,000 lb.

per sq. in. to 35,000 lb. per sq. in. While this variation in the form of the carbon is taking place, the hardness is decreasing from a Brinell hardness of 410 to 130.

The treatment of cast iron during the cooling of the casting very greatly influences the form of the carbon found in the iron. The free carbon comes from the decomposition of the combined carbon, hence any factor which will prevent this decomposition of the combined carbon will result in a harder and stronger iron. Rapid cooling of the casting as well as small amounts of manganese and chromium will help to prevent the formation of free carbon. If a soft weak cast iron is desired, slow cooling and small amounts of silicon and aluminum will tend to favor the decomposition of the combined carbon into free carbon and iron. The rapid cooling is sometimes accomplished by chilling (using cast iron in the surface of the mold in contact with the molten metal).

Small amounts of other chemical elements are also present in cast iron, and exert some influence on the mechanical properties of the material. However, they will not be discussed because the carbon exerts the greatest influence.

There are six principal defects which may occur in cast iron:

1. Coarse grain
2. Segregations
3. Checks
4. Blow holes
5. Cold shut and cold shot
6. Spongy spot

1. Coarse grains in cast iron are large grains which make the material weak and are caused by slow cooling or a high phosphorous content.

2. Segregations in cast iron mean the grouping together of the chemical elements. When such grouping occurs as in the case of iron and sulphur, hard spots are formed in the casting which are weak, and are very difficult to machine.

3. Checks in a casting are small cracks in the surface of the casting due principally to a high sulphur content and improper design of the shape.

4. Blow holes are gas cavities in the casting and are caused by trapped gasses or steam in the mold or in some cases, by a high sulphur content.

5. When a casting is being poured, and the pouring process is interrupted momentarily, the poured metal may solidify before the mold is completely filled. As there is no fusion (bonding together of the two parts of the casting) a plane of weakness, called a cold shut, is formed which seriously affects the strength of the casting. Cold shot are solidified drops (splashings) which are picked up by the molten iron and are found imbedded in, but not fused with, the casting. They, likewise, cause planes of weakness.

6. If the metal solidifies in the risers of a mold before the mold is filled, part of the casting may have a spongy, porous structure which is very weak. This porous part is called a spongy spot.

MALLEABLE (CAST) IRON

When a good grade of white cast iron is packed in powdered iron ore and subjected to a high temperature for a period of time (approximately sixty hours) the combined carbon decomposes into globules of carbon and free iron. The resulting product is termed malleable (cast) iron due to its excellent ductility and tensile strength. Good malleable iron will have a tensile strength of 45,000 to 50,000 lb. per sq. in. which is almost as much as the tensile strength of mild steel. The ductility varies from 5% to 10%, while, for comparison, the white cast iron from which the malleable iron was made would have a relatively small per cent ductility.

NON-FERROUS METALS AND ALLOYS

Copper is an important metal in industry due to its excellent electrical properties and the ease with which it forms alloys with tin, zinc, nickel, and antimony.

When copper is alloyed with zinc in varying percentages, **brass** is formed, the properties of the brass depending on the amounts of the principal metals used. The better brasses contain from 60% to 90% copper with from 40% to 10% zinc. Zinc provides the hardness to the alloy and hence, the soft brasses have smaller amounts of zinc while the hard brasses have larger amounts of zinc.

Brasses which are to be rolled, forged, or extruded contain about 70 parts of copper to 30 parts of zinc and can be hot worked. If more zinc is used, the brass can be cold worked to produce hardness.

The following table gives the mechanical properties of cast brasses:

TABLE 10

% copper	% zinc	Tensile strength, pounds per square inch	% elongation
90	10	30,000	20
80	20	34,000	33
70	30	33,000	38
60	40	52,000	20
50	50	30,000	4
40	60	5,000	0

Bronze is an alloy of copper and tin in which the tin is added to the copper solely as a hardening agent for there is but little increase in tensile strength of the copper itself.

The useful alloys of copper and tin contain from 5 to 25 per cent of tin and 95 to 75 per cent of copper. Bronze has little ductility, fair tensile strength, but for the harder varieties, the compressive strengths increase to a maximum of 150,000 lb. per sq. in. Bronze is highly resistant to corrosion, which property together with its hardness makes the metal very useful.

Certain **complex bronzes,** alloys of copper, tin and zinc show high tensile strengths in their cast form. These alloys, which show a ductility of 20 to 30 per cent, consist of 75 to 85 per cent copper, 17 to 5 per cent zinc, 8 to 10 per cent tin, and have tensile strengths which vary from 35,000 to 40,000 lb. per sq. in.

Aluminum alloys are used in preference to pure aluminum where the requirements demand strength as well as lightness. Thus, aluminum-copper alloys are used for pistons, crank cases, etc. for automobiles. Aluminum alloyed with manganese and copper is used for structural shapes.

Lead, tin, and antimony alloys are rather soft and have low melting temperatures. The lead and tin alloys are used for solder, pewter, and toys while alloys of tin, antimony, lead, and small amounts of copper, are called Babbitt metals and are used for bearings.

Alloys which melt at very low temperatures and are used in fire protection devices contain bismuth in addition to lead, tin, and cadmium. Some of these alloys melt at a temperature as low as 140°F.

Monel Metal is an alloy of about 68% nickel, 2% iron, and 30% copper. In its rolled form, its tensile strength is from 80,000 to 100,000 lb. per sq. in. and it has the ductility of soft steel. Owing to its high cost, its use is rather limited but its high resistance to corrosion makes it valuable for propellers, valves, etc. when exposed to sea water. It is used, also, for kitchen equipment where its bright appearance and durability are important factors.

TIMBER

The two classes of trees which furnish practically all of the timber used in this country are the conifers and the broad leaved trees. These classes are occasionally referred to as the hard woods and the soft woods, but this reference is not exact since yellow pine, a conifer, is a hard wood, and poplar, a broad leaved tree, is a soft wood.

The annual rings of a tree furnish valuable information regarding the age of the tree, the rapidity of growth, the uniformity of growth and the probable location at which the tree was grown. A forest grown tree shows narrow rings except at the top of the tree, and wide rings show growth in the open or in wet ground. In the conifers, a medium rate of growth is preferable while in the broad leaved trees, such as hickory, oak, etc., a wide ring or rapid growth is preferable.

Each year's growth shows two distinct rings, one darker than the other. In the oaks and chestnut, the darker ring is porous and is known as the spring growth while the summer growth is lighter in color and is much harder than the spring growth. In the conifers, the lighter colored ring is the spring growth and is soft compared to the darker colored summer growth. The summer growth is harder and more solid in all trees.

As trees grow, the inner part becomes inactive and is known as the heart wood. This heart wood, which remains firm and strong, furnishes support to the tree. The outer part of the tree, called the sap wood, is the live part. This is the part of the tree between the heartwood and the bark.

From a structural standpoint, there are three important classes of defects in timber:

Knots are the beginning of branches in the parent stem.

Checks are longitudinal cracks in a log, generally in a radial direction.

Shakes are longitudinal cracks in a log, generally between the annual rings.

Drying timber is an important process, for it bears the same relation to the timber as heat-treatment bears to the mechanical properties of steel.

Timber is dried to: decrease the weight; increase its resistance to decay; increase the strength; decrease the shrinkage and prepare it for any preservative treatment.

The best method of drying timber is by a well controlled process of kiln drying. Such a process gives uniform results but is more expensive than air drying or seasoning. One of the results of drying timber too rapidly is known as case hardening, whereby the outer rings dry out and shrink, thereby causing them to split or crack. Continued rapid drying may cause the interior to shrink sufficiently to pull away from the exterior.

Wood Deterioration.—Wood deteriorates rapidly from a number of causes unless treated with preservatives, or preventatives are used. Some of the causes of deterioration are:

Bacteria
Fungi
Insects
Dry rot
Woodpeckers
Fire
Mechanical wear
Moisture

Winter cutting, bark removal, drying, and preventatives will largely overcome the deteriorating causes.

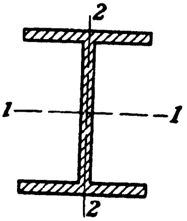
As the engineer is interested in the mechanical properties of timber, the strength being the principal one, he should know the conditions which affect the strength.

The following summary lists the various factors which affect the strength of a piece of timber:

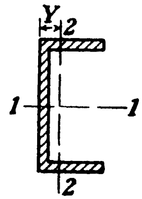
1. Species
2. Rate of growth
3. Location of tree
4. Position in the tree
5. Moisture
6. Defects
7. Deteriorating causes

APPENDIX

AXIS DESIGNATIONS



CB Sections
American Standard Beams.



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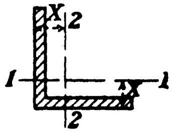

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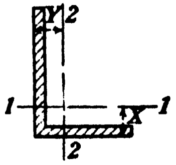

Unequal angles.

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CB SECTIONS

Elements of Sections

Section index and nominal size	Depth of section, in.	Weight per foot, lb.	Area of section, in.^{2}	Flange		Web thickness, in.	Axis 1-1			Axis 2-2		
				Width, in.	Thickness, in.		I, in.^{4}	S, in.^{3}	r, in.	I, in.^{4}	S, in.^{3}	r, in.
CB 362	36.72	300	88.17	16.655	1.680	.945	20,290.2	1105.1	15.17	1225.2	147.1	3.73
36 × 16½	36.24	260	76.56	16.555	1.440	.845	17,233.8	951.1	15.00	1020.6	123.3	3.65
	36.00	240	70.60	16.500	1.320	.790	15,724.0	873.6	14.92	920.1	111.5	3.61
CB 361	36.48	194	57.11	12.117	1.260	.770	12,103.4	663.6	14.56	355.4	58.7	2.49
36 × 12	36.16	170	49.98	12.027	1.100	.680	10,470.0	579.1	14.47	300.6	50.0	2.45
	35.84	150	44.16	11.972	.940	.625	9012.1	502.9	14.29	250.4	41.8	2.38
CB 332	33.50	240	70.52	15.865	1.400	.830	13,585.1	811.1	13.88	874.3	110.2	3.52
33 × 15¾	33.12	210	61.78	15.783	1.210	.748	11,664.5	704.4	13.74	735.6	93.2	3.45
CB 331	33.50	152	44.71	11.565	1.055	.635	8147.6	486.4	13.50	256.1	44.3	2.39
33 × 11½	33.15	132	38.84	11.510	.880	.580	6856.8	413.7	13.29	207.8	36.1	2.31
CB 302	30.38	210	61.78	15.105	1.315	.775	9872.4	649.9	12.64	707.9	93.7	3.38
30 × 15	30.12	190	55.90	15.040	1.185	.710	8825.9	586.1	12.57	624.6	83.1	3.34
	29.88	172	50.65	14.985	1.065	.655	7891.5	528.2	12.48	550.1	73.4	3.30
CB 301	30.30	132	38.83	10.551	1.000	.615	5753.1	379.7	12.17	185.0	35.1	2.18
30 × 10½	30.00	116	34.13	10.500	.850	.564	4919.1	327.9	12.00	153.2	29.2	2.12
CB 272	27.31	177	52.10	14.090	1.190	.725	6728.6	492.8	11.36	518.9	73.7	3.16
27 × 14	27.00	154	45.30	14.000	1.035	.635	5775.8	427.8	11.29	437.6	62.5	3.11
CB 271	27.28	114	33.53	10.070	.932	.570	4080.5	299.2	11.03	149.6	29.7	2.11
27 × 10	27.00	98	28.82	10.000	.792	.500	3446.5	255.3	10.94	122.9	24.6	2.07
CB 243	24.72	160	47.04	14.091	1.135	.656	5110.3	413.5	10.42	492.6	69.9	3.23
24 × 14	24.41	140	41.16	14.029	.980	.594	4376.1	358.6	10.31	414.5	59.1	3.17
CB 242	24.31	120	35.29	12.088	.930	.556	3635.3	299.1	10.15	254.0	42.0	2.68
24 × 12	24.00	100	29.43	12.000	.775	.468	2987.3	248.9	10.08	203.5	33.9	2.63
CB 241	24.29	94	27.63	9.061	.872	.516	2683.0	220.9	9.85	102.2	22.6	1.92
24 × 9	24.00	80	23.54	9.000	.727	.455	2229.7	185.8	9.73	82.4	18.3	1.87
CB 213	21.46	142	41.76	13.132	1.095	.659	3403.1	317.2	9.03	385.9	58.8	3.04
21 × 13	21.16	122	35.85	13.040	.945	.567	2883.2	272.5	8.97	322.1	49.4	3.00
CB 212	21.29	103	30.27	9.071	1.010	.608	2268.0	213.1	8.66	119.9	26.4	1.99
21 × 9	21.00	89	26.15	9.000	.865	.537	1919.2	182.8	8.57	99.4	22.1	1.95
CB 211	21.24	73	21.46	8.295	.740	.455	1600.3	150.7	8.64	66.2	16.0	1.76
21 × 8¼	21.00	63	18.52	8.250	.620	.410	1343.6	128.0	8.52	53.8	13.0	1.70
CB 183	18.64	124	36.45	11.889	1.071	.651	2227.1	239.0	7.82	281.9	47.4	2.78
18 × 11¾	18.32	105	30.86	11.792	.911	.554	1852.5	202.2	7.75	231.0	39.2	2.73
CB 182	18.32	85	24.97	8.838	.911	.526	1429.9	156.1	7.57	99.4	22.5	2.00
18 × 8¾	18.00	70	20.56	8.750	.751	.438	1153.9	128.2	7.49	78.5	17.9	1.95
CB 181	18.12	55	16.19	7.532	.630	.390	889.9	98.2	7.41	42.0	11.1	1.61
18 × 7½	17.90	47	13.81	7.492	.520	.350	736.4	82.3	7.30	33.5	9.0	1.56
CB 163	16.64	114	33.51	11.629	1.035	.631	1642.6	197.4	7.00	254.6	43.8	2.76
16 × 11½	16.32	96	28.22	11.533	.875	.535	1355.1	166.1	6.93	207.2	35.9	2.71
CB 162	16.32	78	22.92	8.586	.875	.529	1042.6	127.8	6.74	87.5	20.4	1.95
16 × 8½	16.00	64	18.80	8.500	.715	.443	833.8	104.2	6.66	68.4	16.1	1.91
CB 161	16.25	50	14.70	7.073	.628	.380	655.4	80.7	6.68	34.8	9.8	1.54
16 × 7	16.00	40	11.77	7.000	.503	.307	515.5	64.4	6.62	26.5	7.6	1.50
	18.69	426	125.25	16.695	3.033	1.875	6610.3	707.4	7.26	2359.5	282.7	4.34
CB 146	18.31	398	116.98	16.590	2.843	1.770	6013.7	656.9	7.17	2169.7	261.6	4.31
14 × 16	17.94	370	108.78	16.475	2.658	1.655	5454.2	608.1	7.08	1986.0	241.1	4.27
	17.56	342	100.59	16.365	2.468	1.545	4911.5	559.4	6.99	1806.9	220.8	4.24

CB SECTIONS
Elements of Sections

Section index and nominal size	Depth of section, in.	Weight per foot, lb.	Area of section, in.^{2}	Flange		Web thickness, in.	Axis 1-1			Axis 2-2		
				Width, in.	Thickness, in.		I, in.^{4}	S, in.^{3}	r, in.	I, in.^{4}	S, in.^{3}	r, in.
CB 146 14 × 16	17.19	314	92.30	16.235	2.283	1.415	4399.4	511.9	6.90	1631.4	201.0	4.20
	16.81	287	84.37	16.130	2.093	1.310	3912.1	465.5	6.81	1466.5	181.8	4.17
	16.50	264	77.63	16.025	1.938	1.205	3526.0	427.4	6.74	1331.2	166.1	4.14
	16.25	246	72.33	15.945	1.813	1.125	3228.9	397.4	6.68	1226.6	153.9	4.12
	16.00	228	67.06	15.865	1.688	1.045	2942.4	367.8	6.62	1124.8	141.8	4.10
	15.75	211	62.07	15.800	1.563	.980	2671.4	339.2	6.56	1028.6	130.2	4.07
	15.50	193	56.73	15.710	1.438	.890	2402.4	310.0	6.51	930.1	118.4	4.05
	15.25	176	51.73	15.640	1.313	.820	2149.6	281.9	6.45	837.9	107.1	4.02
	15.00	158	46.47	15.550	1.188	.730	1900.6	253.4	6.40	745.0	95.8	4.00
	14.75	142	41.85	15.500	1.063	.680	1672.2	226.7	6.32	660.1	85.2	3.97
	16.81	320	94.12	16.710	2.093	1.890	4141.7	492.8	6.63	1635.1	195.7	4.17
CB 145 14 × 14½	14.75	136	39.98	14.740	1.063	.660	1593.0	216.0	6.31	567.7	77.0	3.77
	14.50	119	34.99	14.650	.938	.570	1373.1	189.4	6.26	491.8	67.1	3.75
	14.25	103	30.26	14.575	.813	.495	1165.8	163.6	6.21	419.7	57.6	3.72
	14.00	87	25.56	14.500	.688	.420	966.9	138.1	6.15	349.7	48.2	3.70
CB 144 14 × 12	14.18	84	24.71	12.023	.778	.451	928.4	130.9	6.13	225.5	37.5	3.02
	14.06	78	22.94	12.000	.718	.428	851.2	121.1	6.09	206.9	34.5	3.00
CB 143 14 × 10	14.19	74	21.76	10.072	.783	.450	796.8	112.3	6.05	133.5	26.5	2.48
	13.91	61	17.94	10.000	.643	.378	641.5	92.2	5.98	107.3	21.5	2.45
CB 142 14 × 8	14.06	58	17.06	8.098	.718	.406	597.9	85.0	5.92	63.7	15.7	1.93
	13.81	48	14.11	8.031	.593	.339	484.9	70.2	5.86	51.3	12.8	1.91
CB 141 14 × 6¾	14.24	42	12.34	6.801	.573	.338	432.2	60.7	5.92	28.1	8.3	1.51
	14.00	34	10.00	6.750	.453	.287	339.2	48.5	5.83	21.3	6.3	1.46
CB 124 12 × 12	14.38	190	55.86	12.670	1.736	1.060	1892.5	263.2	5.82	589.7	93.1	3.25
	13.88	161	47.38	12.515	1.486	.905	1541.8	222.2	5.70	486.2	77.7	3.20
	13.38	133	39.11	12.365	1.236	.755	1221.2	182.5	5.59	389.9	63.1	3.16
	12.88	106	31.19	12.230	.986	.620	930.7	144.5	5.46	300.9	49.2	3.11
	12.62	92	27.06	12.155	.856	.545	788.9	125.0	5.40	256.4	42.2	3.08
	12.38	79	23.22	12.080	.736	.470	663.0	107.1	5.34	216.4	35.8	3.05
	12.12	65	19.11	12.000	.606	.390	533.4	88.0	5.28	174.6	29.1	3.02
CB 123 12 × 10	12.31	64	18.83	10.060	.701	.405	528.3	85.8	5.29	119.0	23.7	2.51
	12.06	53	15.59	10.000	.576	.345	426.2	70.7	5.23	96.1	19.2	2.48
CB 122 12 × 8	12.19	50	14.71	8.077	.641	.371	394.5	64.7	5.18	56.4	14.0	1.96
	11.94	40	11.77	8.000	.516	.294	310.1	51.9	5.13	44.1	11.0	1.94
CB 121 12 × 6½	12.24	36	10.59	6.565	.540	.305	280.8	45.9	5.15	23.7	7.2	1.50
	12.00	28	8.23	6.500	.420	.240	213.5	35.6	5.09	17.5	5.4	1.46
CB 103 10 × 10	11.88	136	40.03	10.575	1.498	.915	917.2	154.4	4.79	295.9	56.0	2.72
	11.38	112	32.92	10.415	1.248	.755	718.7	126.3	4.67	235.4	45.2	2.67
	10.88	89	26.19	10.275	.998	.615	542.4	99.7	4.55	180.6	35.2	2.63
	10.50	72	21.18	10.170	.808	.510	420.7	80.1	4.46	141.8	27.9	2.59
	10.25	60	17.66	10.075	.683	.415	343.7	67.1	4.41	116.5	23.1	2.57
	10.00	49	14.40	10.000	.558	.340	272.9	54.6	4.35	93.0	18.6	2.54
CB 102 10 × 8	10.12	45	13.24	8.022	.618	.350	248.6	49.1	4.33	53.2	13.3	2.00
	9.88	37	10.88	7.978	.498	.306	196.9	39.9	4.25	42.2	10.6	1.97
CB 101 10 × 5¾	10.22	29	8.53	5.799	.500	.289	157.3	30.8	4.29	15.2	5.2	1.34
	10.00	23	6.77	5.750	.390	.240	120.6	24.1	4.22	11.3	3.9	1.29
CB 83 8 × 8	9.00	67	19.70	8.287	.933	.575	271.8	60.4	3.71	88.6	21.4	2.12
	8.50	48	14.11	8.117	.683	.405	183.7	43.2	3.61	60.9	15.0	2.08
	8.12	35	10.30	8.027	.493	.315	126.5	31.1	3.50	42.5	10.6	2.03
	8.00	31	9.12	8.000	.433	.288	109.7	27.4	3.47	37.0	9.2	2.01
CB 82 8 × 6½	8.03	27	7.93	6.528	.448	.273	94.1	23.4	3.44	20.8	6.4	1.62
	7.93	24	7.06	6.500	.398	.245	82.5	20.8	3.42	18.2	5.6	1.61
CB 81 8 × 5¼	8.19	21	6.18	5.272	.403	.252	73.8	18.0	3.45	9.13	3.5	1.22
	8.00	17	5.00	5.250	.308	.230	56.4	14.1	3.36	6.72	2.6	1.16

BEAMS, AMERICAN STANDARD
Elements of Sections

Section index and nominal size	Depth of beam, in.	Weight per foot, lb.	Area of section, in.^{2}	Width of flange, in.	Web thickness, in.	Axis 1-1			Axis 2-2		
						I, in.^{4}	S, in.^{3}	r, in.	I, in.^{4}	S, in.^{3}	r, in.
B 18 24 × 7⅞	24	120.0	35.13	8.048	.798	3010.8	250.9	9.26	84.9	21.1	1.56
		115.0	33.67	7.987	.737	2940.5	245.0	9.35	82.8	20.7	1.57
		110.0	32.18	7.925	.675	2869.1	239.1	9.44	80.6	20.3	1.58
		105.9	30.98	7.875	.625	2811.5	234.3	9.53	78.9	20.0	1.60
B 1 24 × 7	24	100.0	29.25	7.247	.747	2371.8	197.6	9.05	48.4	13.4	1.29
		95.0	27.79	7.186	.686	2301.5	191.8	9.08	47.0	13.0	1.30
		90.0	26.30	7.124	.624	2230.1	185.8	9.21	45.5	12.8	1.32
		85.0	24.84	7.063	.563	2159.8	180.0	9.33	44.2	12.5	1.33
		79.9	23.33	7.000	.500	2087.2	173.9	9.46	42.9	12.2	1.36
B 2 20 × 7	20	100.0	29.20	7.273	.873	1648.3	164.8	7.51	52.4	14.4	1.34
		95.0	27.74	7.200	.800	1599.7	160.0	7.59	50.5	14.0	1.35
		90.0	26.26	7.126	.726	1550.3	155.0	7.68	48.7	13.7	1.36
		85.0	24.80	7.053	.653	1501.7	150.2	7.78	47.0	13.3	1.38
		81.4	23.74	7.000	.600	1466.3	146.6	7.86	45.8	13.1	1.39
B 3 20 × 6¼	20	75.0	21.90	6.391	.641	1263.5	126.3	7.60	30.1	9.4	1.17
		70.0	20.42	6.317	.567	1214.2	121.4	7.71	28.9	9.2	1.19
		65.4	19.08	6.250	.500	1169.5	116.9	7.83	27.9	8.9	1.21
B 4 18 × 6	18	70.0	20.46	6.251	.711	917.5	101.9	6.70	24.5	7.8	1.09
		65.0	18.98	6.169	.629	877.7	97.5	6.80	23.4	7.6	1.11
		60.0	17.50	6.087	.547	837.8	93.1	6.92	22.3	7.3	1.13
		54.7	15.94	6.000	.460	795.5	88.4	7.07	21.2	7.1	1.15
B 6 15 × 6	15	75.0	21.85	6.278	.868	687.2	91.6	5.61	30.6	9.8	1.18
		70.0	20.38	6.180	.770	659.6	87.9	5.69	28.8	9.3	1.19
		65.0	18.91	6.082	.672	632.1	84.3	5.78	27.2	8.9	1.20
		60.8	17.68	6.000	.590	609.0	81.2	5.87	26.0	8.7	1.21
B 7 15 × 5½	15	55.0	16.06	5.738	.648	508.7	67.8	5.63	17.0	5.9	1.03
		50.0	14.59	5.640	.550	481.1	64.2	5.74	16.0	5.7	1.05
		45.0	13.12	5.542	.452	453.6	60.5	5.88	15.0	5.4	1.07
		42.9	12.49	5.500	.410	441.8	58.9	5.95	14.6	5.3	1.08
B 8 12 × 5¼	12	55.0	16.04	5.600	.810	319.3	53.2	4.46	17.3	6.2	1.04
		50.0	14.57	5.477	.687	301.6	50.3	4.55	16.0	5.8	1.05
		45.0	13.10	5.355	.565	284.1	47.3	4.66	14.8	5.5	1.06
		40.8	11.84	5.250	.460	268.9	44.8	4.77	13.8	5.3	1.08
B 9 12 × 5	12	35.0	10.20	5.078	.428	227.0	37.8	4.72	10.0	3.9	0.99
		31.8	9.26	5.000	.350	215.8	36.0	4.83	9.5	3.8	1.01
B 10 10 × 4⅝	10	40.0	11.69	5.091	.741	158.0	31.6	3.68	9.4	3.7	0.90
		35.0	10.22	4.944	.594	145.8	29.2	3.78	8.5	3.4	0.91
		30.0	8.75	4.797	.447	133.5	26.7	3.91	7.6	3.2	0.93
		25.4	7.38	4.660	.310	122.1	24.4	4.07	6.9	3.0	0.97

BEAMS, AMERICAN STANDARD
Elements of Sections

Section index and nominal size	Depth of beam, in.	Weight per foot, lb.	Area of section, in.^{2}	Width of flange, in.	Web thickness, in.	Axis 1-1			Axis 2-2		
						I, in.^{4}	S, in.^{3}	r, in.	I, in.^{4}	S, in.^{3}	r, in.
		25.5	7.43	4.262	.532	68.1	17.0	3.03	4.7	2.2	0.80
B 12 8 × 4	8	23.0	6.71	4.171	.441	64.2	16.0	3.09	4.4	2.1	0.81
		20.5	5.97	4.079	.349	60.2	15.1	3.18	4.0	2.0	0.82
		18.4	5.34	4.000	.270	56.9	14.2	3.26	3.8	1.9	0.84
		20.0	5.83	3.860	.450	41.9	12.0	2.68	3.1	1.6	0.74
B 13 7 × 3⅝	7	17.5	5.09	3.755	.345	38.9	11.1	2.77	2.9	1.6	0.76
		15.3	4.43	3.660	.250	36.2	10.4	2.86	2.7	1.5	0.78
		17.25	5.02	3.565	.465	26.0	8.7	2.28	2.3	1.3	0.68
B 14 6 × 3⅜	6	14.75	4.29	3.443	.343	23.8	7.9	2.36	2.1	1.2	0.69
		12.5	3.61	3.330	.230	21.8	7.3	2.46	1.8	1.1	0.72
		14.75	4.29	3.284	.494	15.0	6.0	1.87	1.7	1.0	0.63
B 15 5 × 3	5	12.25	3.56	3.137	.347	13.5	5.4	1.95	1.4	0.91	0.63
		10.0	2.87	3.000	.210	12.1	4.8	2.05	1.2	0 82	0.65
		10.5	3.05	2.870	.400	7.1	3.5	1.52	1.0	0.70	0.57
B 16 4 × 2⅝	4	9.5	2.76	2.796	.326	6.7	3.3	1.56	0.91	0.65	0.58
		8.5	2.46	2.723	.253	6.3	3.2	1.60	0.83	0.61	0.58
		7.7	2.21	2.660	.190	6.0	3.0	1.64	0.77	0.58	0.59
		7.5	2.17	2.509	.349	2.9	1.9	1.15	0.59	0.47	0.52
B 17 3 × 2⅜	3	6.5	1.88	2.411	.251	2.7	1.8	1.19	0.51	0.43	0.52
		5.7	1.64	2.330	.170	2.5	1.7	1.23	0.46	0.40	0.53

CHANNELS, AMERICAN STANDARD
Elements of Sections

Section index and nominal size	Depth of channel, in.	Weight per foot, lb.	Area of section, in.^{2}	Width of flange, in.	Web thickness, in.	Axis 1-1			Axis 2-2			
						I, in.^{4}	S, in.^{3}	r, in.	I, in.^{4}	S, in.^{3}	r, in.	y, in.
C 60 18 × 4	18	58.0	16.98	4.200	.700	670.7	74.5	6.29	18.5	5.6	1.04	0.88
		51.9	15.18	4.100	.600	622.1	69.1	6.40	17.1	5.3	1.06	0.87
		45.8	13.38	4.000	.500	573.5	63.7	6.55	15.8	5.1	1.09	0.89
		42.7	12.48	3.950	.450	549.2	61.0	6.64	15.0	4.9	1.10	0.90
C 1 15 × 3⅜	15	55.0	16.11	3.814	.814	429.0	57.2	5.16	12.1	4.1	0.87	0.82
		50.0	14.64	3.716	.716	401.4	53.6	5.24	11.2	3.8	0.87	0.80
		45.0	13.17	3.618	.618	373.9	49.8	5.33	10.3	3.6	0.88	0.79
		40.0	11.70	3.520	.520	346.3	46.2	5.44	9.3	3.4	0.89	0.78
		35.0	10.23	3.422	.422	318.7	42.5	5.58	8.4	3.2	0.91	0.79
		33.9	9.90	3.400	.400	312.6	41.7	5.62	8.2	3.2	0.91	0.79
C 2 12 × 3	12	40.0	11.73	3.415	.755	196.5	32.8	4.09	6.6	2.5	0.75	0.72
		35.0	10.26	3.292	.632	178.8	29.8	4.18	5.9	2.3	0.76	0.69
		30.0	8.79	3.170	.510	161.2	26.9	4.28	5.2	2.1	0.77	0.68
		25.0	7.32	3.047	.387	143.5	23.9	4.43	4.5	1.9	0.79	0.68
		20.7	6.03	2.940	.280	128.1	21.4	4.61	3.9	1.7	0.81	0.70
C 3 10 × 2⅝	10	35.0	10.27	3.180	.820	115.2	23.0	3.34	4.6	1.9	0.67	0.69
		30.0	8.80	3.033	.673	103.0	20.6	3.42	4.0	1.7	0.67	0.65
		25.0	7.33	2.886	.526	90.7	18.1	3.52	3.4	1.5	0.68	0.62
		20.0	5.86	2.739	.379	78.5	15.7	3.66	2.8	1.3	0.70	0.61
		15.3	4.47	2.600	.240	66.9	13.4	3.87	2.3	1.2	0.72	0.64
C 4 9 × 2½	9	25.0	7.33	2.812	.612	70.5	15.7	3.10	3.0	1.4	0.64	0.61
		20.0	5.86	2.648	.448	60.6	13.5	3.22	2.4	1.2	0.65	0.59
		15.0	4.39	2.485	.285	50.7	11.3	3.40	1.9	1.0	0.67	0.59
		13.4	3.89	2.430	.230	47.3	10.5	3.49	1.8	0.97	0.67	0.61
C 5 8 × 2¼	8	21.25	6.23	2.619	.579	47.6	11.9	2.77	2.2	1.1	0.60	0.59
		18.75	5.49	2.527	.487	43.7	10.9	2.82	2.0	1.0	0.60	0.57
		16.25	4.76	2.435	.395	39.8	9.9	2.89	1.8	0.94	0.61	0.56
		13.75	4.02	2.343	.303	35.8	9.0	2.99	1.5	0.86	0.62	0.56
		11.5	3.36	2.260	.220	32.3	8.1	3.10	1.3	0.79	0.63	0.58
C 6 7 × 2	7	19.75	5.79	2.509	.629	33.1	9.4	2.39	1.8	0.96	0.56	0.58
		17.25	5.05	2.404	.524	30.1	8.6	2.44	1.6	0.86	0.56	0.55
		14.75	4.32	2.299	.419	27.1	7.7	2.51	1.4	0.79	0.57	0.53
		12.25	3.58	2.194	.314	24.1	6.9	2.59	1.2	0.71	0.58	0.53
		9.8	2.85	2.000	.210	21.1	6.0	2.72	0.98	0.63	0.59	0.55
C 7 6 × 2	6	15.5	4.54	2.279	.559	19.5	6.5	2.07	1.3	0.73	0.53	0.55
		13.0	3.81	2.157	.437	17.3	5.8	2.13	1.1	0.65	0.53	0.52
		10.5	3.07	2.034	.314	15.1	5.0	2.22	0.87	0.57	0.53	0.50
		8.2	2.39	1.920	.200	13.0	4.3	2.34	0.70	0.50	0.54	0.52
C 8 5 × 1¾	5	11.5	3.36	2.032	.472	10.4	4.1	1.76	0.82	0.54	0.49	0.51
		9.0	2.63	1.885	.325	8.8	3.5	1.83	0.64	0.45	0.49	0.48
		6.7	1.95	1.750	.190	7.4	3.0	1.95	0.48	0.38	0.50	0.49
C 9 4 × 1⅝	4	7.25	2.12	1.720	.320	4.5	2.3	1.47	0.44	0.35	0.46	0.46
		6.25	1.82	1.647	.247	4.1	2.1	1.50	0.38	0.32	0.45	0.46
		5.4	1.56	1.580	.180	3.8	1.9	1.56	0.32	0.29	0.45	0.46
C 10 3 × 1½	3	6.0	1.75	1.596	.356	2.1	1.4	1.08	0.31	0.27	0.42	0.46
		5.0	1.46	1.498	.258	1.8	1.2	1.12	0.25	0.24	0.41	0.44
		4.1	1.19	1.410	.170	1.6	1.1	1.17	0.20	0.21	0.41	0.44

EQUAL ANGLES
Elements of Sections

Section index	Size, in.	Thickness, in.	Weight per foot, lb.	Area of section, in.^{2}	Axis 1-1 and axis 2-2			
					I, in.^{4}	S, in.^{3}	r, in.	x, in.
A 1	8 × 8	$1\frac{1}{8}$	56.9	16.73	98.0	17.5	2.42	2.41
		$1\frac{1}{16}$	54.0	15.87	93.5	16.7	2.43	2.39
		1	51.0	15.00	89.0	15.8	2.44	2.37
		$\frac{15}{16}$	48.1	14.12	84.3	14.9	2.44	2.34
		$\frac{7}{8}$	45.0	13.23	79.6	14.0	2.45	2.32
		$\frac{13}{16}$	42.0	12.34	74.7	13.1	2.46	2.30
		$\frac{3}{4}$	38.9	11.44	69.7	12.2	2.47	2.28
		$\frac{11}{16}$	35.8	10.53	64.6	11.2	2.48	2.25
		$\frac{5}{8}$	32.7	9.61	59.4	10.3	2.49	2.23
		$\frac{9}{16}$	29.6	8.68	54.1	9.3	2.50	2.21
		$\frac{1}{2}$	26.4	7.75	48.6	8.4	2.51	2.19
A 2	6 × 6	$1\frac{1}{16}$	39.6	11.62	37.2	9.0	1.79	1.89
		1	37.4	11.00	35.5	8.6	1.80	1.86
		$\frac{15}{16}$	35.3	10.37	33.7	8.1	1.80	1.84
		$\frac{7}{8}$	33.1	9.73	31.9	7.6	1.81	1.82
		$\frac{13}{16}$	31.0	9.09	30.1	7.2	1.82	1.80
		$\frac{3}{4}$	28.7	8.44	28.2	6.7	1.83	1.78
		$\frac{11}{16}$	26.5	7.78	26.2	6.2	1.83	1.75
		$\frac{5}{8}$	24.2	7.11	24.2	5.7	1.84	1.73
		$\frac{9}{16}$	21.9	6.43	22.1	5.1	1.85	1.71
		$\frac{1}{2}$	19.6	5.75	19.9	4.6	1.86	1.68
		$\frac{7}{16}$	17.2	5.06	17.7	4.1	1.87	1.66
		$\frac{3}{8}$	14.9	4.36	15.4	3.5	1.88	1.64
A 3	5 × 5	1	30.6	9.00	19.6	5.8	1.48	1.61
		$\frac{15}{16}$	28.9	8.50	18.7	5.5	1.48	1.59
		$\frac{7}{8}$	27.2	7.98	17.8	5.2	1.49	1.57
		$\frac{13}{16}$	25.4	7.47	16.8	4.9	1.50	1.55
		$\frac{3}{4}$	23.6	6.94	15.7	4.5	1.50	1.52
		$\frac{11}{16}$	21.8	6.40	14.7	4.2	1.51	1.50
		$\frac{5}{8}$	20.0	5.86	13.6	3.9	1.52	1.48
		$\frac{9}{16}$	18.1	5.31	12.4	3.5	1.53	1.46
		$\frac{1}{2}$	16.2	4.75	11.3	3.2	1.54	1.43
		$\frac{7}{16}$	14.3	4.18	10.0	2.8	1.55	1.41
		$\frac{3}{8}$	12.3	3.61	8.7	2.4	1.56	1.39
A 4	4 × 4	$\frac{13}{16}$	19.9	5.84	8.1	3.0	1.18	1.29
		$\frac{3}{4}$	18.5	5.44	7.7	2.8	1.19	1.27
		$\frac{11}{16}$	17.1	5.03	7.2	2.6	1.19	1.25
		$\frac{5}{8}$	15.7	4.61	6.7	2.4	1.20	1.23
		$\frac{9}{16}$	14.3	4.18	6.1	2.2	1.21	1.21
		$\frac{1}{2}$	12.8	3.75	5.6	2.0	1.22	1.18
		$\frac{7}{16}$	11.3	3.31	5.0	1.8	1.23	1.16
		$\frac{3}{8}$	9.8	2.86	4.4	1.5	1.23	1.14
		$\frac{5}{16}$	8.2	2.40	3.7	1.3	1.24	1.12
		$\frac{1}{4}$	6.6	1.94	3.0	1.0	1.25	1.09

EQUAL ANGLES
Elements of Sections

Section index	Size, in.	Thickness, in.	Weight per foot, lb.	Area of section, in.^{2}	Axis 1-1 and axis 2-2			
					I, in.^{4}	S, in.^{3}	r, in.	x, in.
A 5	3½ × 3½	13/16	17.1	5.03	5.3	2.3	1.02	1.17
		3/4	16.0	4.69	5.0	2.1	1.03	1.15
		11/16	14.8	4.34	4.7	2.0	1.04	1.12
		5/8	13.6	3.98	4.3	1.8	1.04	1.10
		9/16	12.4	3.62	4.0	1.6	1.05	1.08
		1/2	11.1	3.25	3.6	1.5	1.06	1.08
		7/16	9.8	2.87	3.3	1.3	1.07	1.04
		3/8	8.5	2.48	2.9	1.2	1.07	1.01
		5/16	7.2	2.09	2.5	0.98	1.08	0.99
		1/4	5.8	1.69	2.0	0.79	1.09	0.97
A 7	3 × 3	5/8	11.5	3.36	2.6	1.3	0.88	0.98
		9/16	10.4	3.06	2.4	1.2	0.89	0.95
		1/2	9.4	2.75	2.2	1.1	0.90	0.93
		7/16	8.3	2.43	2.0	0.95	0.91	0.91
		3/8	7.2	2.11	1.8	0.83	0.91	0.89
		5/16	6.1	1.78	1.5	0.71	0.92	0.87
		1/4	4.9	1.44	1.2	0.58	0.93	0.84
A 9	2½ × 2½	1/2	7.7	2.25	1.2	0.73	0.74	0.81
		7/16	6.8	2.00	1.1	0.65	0.75	0.78
		3/8	5.9	1.73	0.98	0.57	0.75	0.76
		5/16	5.0	1.47	0.85	0.48	0.76	0.74
		1/4	4.1	1.19	0.70	0.39	0.77	0.72
		3/16	3.07	0.90	0.55	0.30	0.78	0.69
		1/8	2.08	0.61	0.38	0.20	0.79	0.67
A 11	2 × 2	7/16	5.3	1.56	0.54	0.40	0.59	0.66
		3/8	4.7	1.36	0.48	0.35	0.59	0.64
		5/16	3.92	1.15	0.42	0.30	0.60	0.61
		1/4	3.19	0.94	0.35	0.25	0.61	0.59
		3/16	2.44	0.71	0.28	0.19	0.62	0.57
		1/8	1.65	0.48	0.19	0.13	0.63	0.55
A 12	1¾ × 1¾	5/16	3.39	1.00	0.27	0.23	0.52	0.55
		1/4	2.77	0.81	0.23	0.19	0.53	0.53
		3/16	2.12	0.62	0.18	0.14	0.54	0.51
		1/8	1.44	0.42	0.13	0.10	0.55	0.48
A 13	1½ × 1½	3/8	3.35	0.98	0.19	0.19	0.44	0.51
		5/16	2.86	0.84	0.16	0.16	0.44	0.49
		1/4	2.34	0.69	0.14	0.13	0.45	0.47
		3/16	1.80	0.53	0.11	0.10	0.46	0.44
		1/8	1.23	0.36	0.08	0.07	0.46	0.42
A 15	1¼ × 1¼	5/16	2.33	0.68	0.09	0.11	0.36	0.42
		1/4	1.92	0.56	0.08	0.09	0.37	0.40
		3/16	1.48	0.43	0.06	0.07	0.38	0.38
		1/8	1.01	0.30	0.04	0.05	0.38	0.35
A 16	1 × 1	1/4	1.49	0.44	0.04	0.06	0.29	0.34
		3/16	1.16	0.34	0.03	0.04	0.30	0.32
		1/8	0.80	0.23	0.02	0.08	0.31	0.30

UNEQUAL ANGLES
Elements of Sections

Section index	Size, in.	Thickness, in.	Weight per foot, lb.	Area of section, in.^{2}	Axis 1-1				Axis 2-2			
					I, in.^{4}	S, in.^{3}	r, in.	x, in.	I, in.^{4}	S, in.^{3}	r, in.	y, in.
A 18	8 × 6	1 1/8	49.3	14.48	88.9	16.8	2.48	2.70	42.5	9.9	1.71	1.70
		1 1/16	46.8	13.75	84.9	15.9	2.48	2.68	40.7	9.4	1.72	1.68
		1	44.2	13.00	80.8	15.1	2.49	2.65	38.8	8.9	1.73	1.65
		15/16	41.7	12.25	76.6	14.3	2.50	2.63	36.8	8.4	1.73	1.63
		7/8	39.1	11.48	72.3	13.4	2.51	2.61	34.9	7.9	1.74	1.61
		13/16	36.5	10.72	67.9	12.5	2.52	2.59	32.8	7.4	1.75	1.59
		3/4	33.8	9.94	63.4	11.7	2.53	2.56	30.7	6.9	1.76	1.56
		11/16	31.2	9.15	58.8	10.8	2.54	2.54	28.6	6.4	1.77	1.54
		5/8	28.5	8.36	54.1	9.9	2.54	2.52	26.3	5.9	1.77	1.52
		9/16	25.7	7.56	49.3	8.9	2.55	2.50	24.0	5.3	1.78	1.50
		1/2	23.0	6.75	44.3	8.0	2.56	2.47	21.7	4.8	1.79	1.47
		7/16	20.2	5.93	39.2	7.1	2.57	2.45	19.3	4.2	1.80	1.45
A 50	8 × 4	1	37.4	11.00	69.6	14.1	2.52	3.05	11.6	3.9	1.03	1.05
		15/16	35.3	10.37	66.1	13.3	2.52	3.02	11.1	3.7	1.03	1.02
		7/8	33.1	9.73	62.4	12.5	2.53	3.00	10.5	3.5	1.04	1.00
		13/16	31.0	9.09	58.7	11.7	2.54	2.98	10.0	3.3	1.05	0.98
		3/4	28.7	8.44	54.9	10.9	2.55	2.95	9.4	3.1	1.05	0.95
		11/16	26.5	7.78	51.0	10.0	2.56	2.93	8.7	2.8	1.06	0.93
		5/8	24.2	7.11	46.9	9.2	2.56	2.91	8.1	2.6	1.07	0.91
		9/16	21.9	6.43	42.8	8.4	2.58	2.88	7.4	2.4	1.07	0.88
		1/2	19.6	5.75	38.5	7.5	2.59	2.86	6.7	2.2	1.08	0.86
		7/16	17.2	5.06	34.1	6.6	2.60	2.83	6.0	1.9	1.09	0.83
A 60	7 × 4	1	34.0	10.00	47.7	10.8	2.18	2.60	11.2	3.9	1.06	1.10
		15/16	32.1	9.44	45.4	10.3	2.19	2.58	10.7	3.7	1.07	1.08
		7/8	30.2	8.86	42.9	9.7	2.20	2.55	10.2	3.5	1.07	1.05
		13/16	28.2	8.28	40.4	9.0	2.21	2.53	9.6	3.2	1.08	1.03
		3/4	26.2	7.69	37.8	8.4	2.22	2.51	9.1	3.0	1.09	1.01
		11/16	24.2	7.09	35.1	7.8	2.23	2.49	8.5	2.8	1.09	0.99
		5/8	22.1	6.49	32.4	7.1	2.24	2.46	7.8	2.6	1.10	0.96
		9/16	20.0	5.88	29.6	6.5	2.24	2.44	7.2	2.4	1.11	0.94
		1/2	17.9	5.25	26.7	5.8	2.25	2.42	6.5	2.1	1.11	0.92
		7/16	15.8	4.63	23.7	5.1	2.26	2.39	5.8	1.9	1.12	0.89
		3/8	13.6	3.99	20.6	4.4	2.27	2.37	5.1	1.6	1.13	0.87
A 20	6 × 4	1	30.6	9.00	30.8	8.0	1.85	2.17	10.8	3.8	1.09	1.17
		15/16	28.9	8.50	29.3	7.6	1.86	2.14	10.3	3.6	1.10	1.14
		7/8	27.2	7.98	27.7	7.2	1.86	2.12	9.8	3.4	1.11	1.12
		13/16	25.4	7.47	26.1	6.7	1.87	2.10	9.2	3.2	1.11	1.10
		3/4	23.6	6.94	24.5	6.2	1.88	2.08	8.7	3.0	1.12	1.08
		11/16	21.8	6.40	22.8	5.8	1.89	2.06	8.1	2.8	1.13	1.06
		5/8	20.0	5.86	21.1	5.3	1.90	2.03	7.5	2.5	1.13	1.03
		9/16	18.1	5.31	19.3	4.8	1.90	2.01	6.9	2.3	1.14	1.01
		1/2	16.2	4.75	17.4	4.3	1.91	1.99	6.3	2.1	1.15	0.99
		7/16	14.3	4.18	15.5	3.8	1.92	1.96	5.6	1.8	1.16	0.96
		3/8	12.3	3.61	13.5	3.3	1.93	1.94	4.9	1.6	1.17	0.94
A 23	5 × 3½	7/8	22.7	6.67	15.7	4.9	1.53	1.79	6.2	2.5	0.96	1.04
		13/16	21.3	6.25	14.8	4.6	1.54	1.77	5.9	2.4	0.97	1.02
		3/4	19.8	5.81	13.9	4.3	1.55	1.75	5.6	2.2	0.98	1.00
		11/16	18.3	5.37	13.0	4.0	1.56	1.72	5.2	2.1	0.98	0.97
		5/8	16.8	4.92	12.0	3.7	1.56	1.70	4.8	1.9	0.99	0.95
		9/16	15.2	4.47	11.0	3.3	1.57	1.68	4.4	1.7	1.00	0.93
		1/2	13.6	4.00	10.0	3.0	1.58	1.66	4.0	1.6	1.01	0.91
		7/16	12.0	3.53	8.9	2.6	1.59	1.63	3.6	1.4	1.01	0.88
		3/8	10.4	3.05	7.8	2.3	1.60	1.61	3.2	1.2	1.02	0.86
		5/16	8.7	2.56	6.6	1.9	1.61	1.59	2.7	1.0	1.03	0.84
A 26	4 × 3½	13/16	18.5	5.43	7.8	2.9	1.19	1.36	5.5	2.3	1.01	1.11
		3/4	17.3	5.06	7.3	2.8	1.20	1.34	5.2	2.1	1.01	1.09
		11/16	16.0	4.68	6.9	2.6	1.21	1.32	4.9	2.0	1.02	1.07
		5/8	14.7	4.30	6.4	2.4	1.22	1.29	4.5	1.8	1.03	1.04
		9/16	13.3	3.90	5.9	2.1	1.23	1.27	4.2	1.7	1.03	1.02
		1/2	11.9	3.50	5.3	1.9	1.23	1.25	3.8	1.5	1.04	1.00
		7/16	10.6	3.09	4.8	1.7	1.24	1.23	3.4	1.3	1.05	0.98
		3/8	9.1	2.67	4.2	1.5	1.25	1.21	3.0	1.2	1.06	0.96
		5/16	7.7	2.25	3.6	1.3	1.26	1.18	2.6	1.0	1.07	0.93

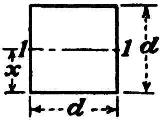
UNEQUAL ANGLES
Elements of Sections

Section index	Size, in.	Thickness, in.	Weight per foot, lb.	Area of section, in.^{2}	Axis 1-1				Axis 2-2			
					I, in.^{4}	S, in.^{3}	r, in.	x, in.	I, in.^{4}	S, in.^{3}	r, in.	y, in.
A 27	4 × 3	$\frac{13}{16}$	17.1	5.03	7.3	2.9	1.21	1.44	3.5	1.7	0.83	0.94
		$\frac{3}{4}$	16.0	4.69	6.9	2.7	1.22	1.42	3.3	1.6	0.84	0.92
		$\frac{11}{16}$	14.8	4.34	6.5	2.5	1.22	1.39	3.1	1.5	0.84	0.89
		$\frac{5}{8}$	13.6	3.98	6.0	2.3	1.23	1.37	2.9	1.4	0.85	0.87
		$\frac{9}{16}$	12.4	3.62	5.6	2.1	1.24	1.35	2.7	1.2	0.86	0.85
		$\frac{1}{2}$	11.1	3.25	5.0	1.9	1.25	1.33	2.4	1.1	0.86	0.83
		$\frac{7}{16}$	9.8	2.87	4.5	1.7	1.25	1.30	2.2	1.0	0.87	0.80
		$\frac{3}{8}$	8.5	2.48	4.0	1.5	1.26	1.28	1.9	0.87	0.88	0.78
		$\frac{5}{16}$	7.2	2.09	3.4	1.2	1.27	1.26	1.7	0.74	0.89	0.76
		$\frac{1}{4}$	5.8	1.69	2.8	1.0	1.28	1.24	1.4	0.60	0.89	0.74
A 28	3½ × 3	$\frac{13}{16}$	15.8	4.62	5.0	2.2	1.04	1.23	3.3	1.7	0.85	0.98
		$\frac{3}{4}$	14.7	4.31	4.7	2.1	1.04	1.21	3.1	1.5	0.85	0.96
		$\frac{11}{16}$	13.6	4.00	4.4	1.9	1.05	1.19	3.0	1.4	0.86	0.94
		$\frac{5}{8}$	12.5	3.67	4.1	1.8	1.06	1.17	2.8	1.3	0.87	0.92
		$\frac{9}{16}$	11.4	3.34	3.8	1.6	1.07	1.15	2.5	1.2	0.87	0.90
		$\frac{1}{2}$	10.2	3.00	3.5	1.5	1.07	1.13	2.3	1.1	0.88	0.88
		$\frac{7}{16}$	9.1	2.65	3.1	1.3	1.08	1.10	2.1	0.98	0.89	0.85
		$\frac{3}{8}$	7.9	2.30	2.7	1.1	1.09	1.08	1.8	0.85	0.90	0.83
		$\frac{5}{16}$	6.6	1.93	2.3	0.96	1.10	1.06	1.6	0.72	0.90	0.81
		$\frac{1}{4}$	5.4	1.56	1.9	0.78	1.11	1.04	1.3	0.58	0.91	0.79
A 32	3 × 2½	$\frac{9}{16}$	9.5	2.78	2.3	1.2	0.91	1.02	1.4	0.82	0.72	0.77
		$\frac{1}{2}$	8.5	2.50	2.1	1.0	0.91	1.00	1.3	0.74	0.72	0.75
		$\frac{7}{16}$	7.6	2.21	1.9	0.93	0.92	0.98	1.2	0.66	0.73	0.73
		$\frac{3}{8}$	6.6	1.92	1.7	0.81	0.93	0.96	1.0	0.58	0.74	0.71
		$\frac{5}{16}$	5.6	1.62	1.4	0.69	0.94	0.93	0.90	0.49	0.74	0.68
		$\frac{1}{4}$	4.5	1.31	1.2	0.56	0.95	0.91	0.74	0.40	0.75	0.66
		$\frac{3}{16}$	3.39	1.00	0.91	0.43	0.95	0.89	0.58	0.31	0.76	0.64
A 35	2½ × 2	$\frac{1}{2}$	6.8	2.00	1.1	0.70	0.75	0.88	0.64	0.46	0.56	0.63
		$\frac{7}{16}$	6.1	1.78	1.0	0.62	0.76	0.85	0.58	0.41	0.68	0.60
		$\frac{3}{8}$	5.3	1.55	0.91	0.55	0.77	0.83	0.51	0.36	0.58	0.58
		$\frac{5}{16}$	4.5	1.31	0.79	0.47	0.78	0.81	0.45	0.31	0.58	0.56
		$\frac{1}{4}$	3.62	1.06	0.65	0.38	0.78	0.79	0.37	0.25	0.59	0.54
		$\frac{3}{16}$	2.75	0.81	0.51	0.29	0.79	0.76	0.29	0.20	0.60	0.51
		$\frac{1}{8}$	1.86	0.55	0.35	0.20	0.80	0.74	0.20	0.13	0.61	0.49
A 37	2 × 1½	$\frac{3}{8}$	3.99	1.17	0.43	0.34	0.61	0.71	0.21	0.20	0.42	0.46
		$\frac{5}{16}$	3.39	1.00	0.38	0.29	0.62	0.69	0.18	0.17	0.42	0.44
		$\frac{1}{4}$	2.77	0.81	0.32	0.24	0.62	0.66	0.15	0.14	0.43	0.41
		$\frac{3}{16}$	2.12	0.62	0.25	0.18	0.63	0.64	0.12	0.11	0.44	0.39
		$\frac{1}{8}$	1.44	0.42	0.17	0.13	0.64	0.62	0.09	0.08	0.45	0.37
A 39	1¾ × 1¼	$\frac{1}{4}$	2.34	0.69	0.20	0.18	0.54	0.60	0.09	0.10	0.35	0.35
		$\frac{3}{16}$	1.80	0.53	0.16	0.14	0.55	0.58	0.07	0.08	0.36	0.33
		$\frac{1}{8}$	1.23	0.36	0.11	0.09	0.56	0.56	0.05	0.05	0.37	0.31

Elements of Sections

SQUARE

Axis of moments through center



$$A = d^2$$

$$x = \frac{d}{2}$$

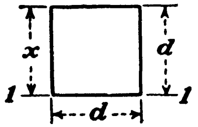
$$I_{1\text{-}1} = \frac{d^4}{12}$$

$$S_{1\text{-}1} = \frac{d^3}{6}$$

$$r_{1\text{-}1} = \frac{d}{\sqrt{12}} = 0.288675d$$

SQUARE

Axis of moments on base



$$A = d^2$$

$$x = d$$

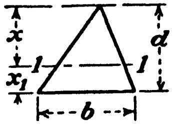
$$I_{1\text{-}1} = \frac{d^4}{3}$$

$$S_{1\text{-}1} = \frac{d^3}{3}$$

$$r_{1\text{-}1} = \frac{d}{\sqrt{3}} = 0.577350d$$

TRIANGLE

Axis of moments through center of gravity



$$A = \frac{bd}{2}$$

$$x = \frac{2d}{3}$$

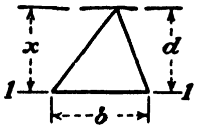
$$I_{1\text{-}1} = \frac{bd^3}{36}$$

$$S_{1\text{-}1} = \frac{bd^2}{24}$$

$$r_{1\text{-}1} = \frac{d}{\sqrt{18}} = 0.235702d$$

TRIANGLE

Axis of moments on base



$$A = \frac{bd}{2}$$

$$x = 2$$

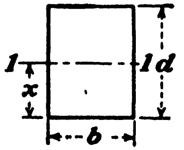
$$I_{1\text{-}1} = \frac{bd^3}{12}$$

$$S_{1\text{-}1} = \frac{bd^2}{12}$$

$$r_{1\text{-}1} = \frac{d}{\sqrt{6}} = 0.408248d$$

RECTANGLE

Axis of moments through center



$$A = bd$$

$$x = \frac{d}{2}$$

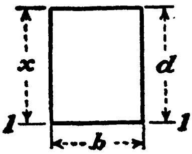
$$I_{1\text{-}1} = \frac{bd^3}{12}$$

$$S_{1\text{-}1} = \frac{bd^2}{6}$$

$$r_{1\text{-}1} = \frac{d}{\sqrt{12}} = 0.288675d$$

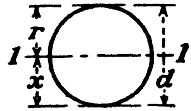
ELEMENTS OF SECTIONS

RECTANGLE
Axis of moments on base



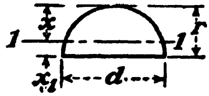
$$A = bd$$
$$x = d$$
$$I_{1\text{-}1} = \frac{bd^3}{3}$$
$$S_{1\text{-}1} = \frac{bd^2}{3}$$
$$r_{1\text{-}1} = \frac{d}{\sqrt{3}} = 0.577350d$$

CIRCLE
Axis of moments through center

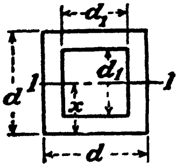


$$A = \frac{\pi d^2}{4} = \pi r^2 = 0.78540d^2 = 3.14159r^2$$
$$x = \frac{d}{2} = r$$
$$I_{1\text{-}1} = \frac{\pi d^4}{64} = \frac{\pi r^4}{4} = 0.04909d^4 = 0.78540r^4$$
$$S_{1\text{-}1} = \frac{\pi d^3}{32} = \frac{\pi r^3}{4} = 0.09818d^3 = 0.78540r^3$$
$$r_{1\text{-}1} = \frac{d}{4} = \frac{r}{2}$$

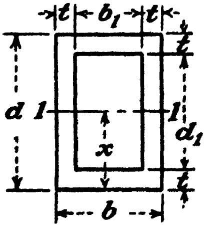
HALF CIRCLE
Axis of moments through center of gravity



$$A = \frac{\pi r^2}{2} = 1.57080r^2$$
$$x = r\left(1 - \frac{4}{3\pi}\right) = 0.57559r \quad x_1 = \frac{4r}{3\pi} = 0.42441r$$
$$I_{1\text{-}1} = r^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) = 0.10976r^4$$
$$S_{1\text{-}1} = \frac{r^3}{24}\,\frac{(9\pi^2 - 64)}{(3\pi - 4)} = 0.19069r^3$$
$$r_{1\text{-}1} = r\,\frac{\sqrt{9\pi^2 - 64}}{6\pi} = 0.26434r$$

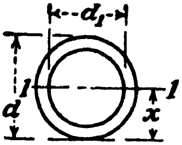


$$A = d^2 - d_1^2$$
$$x = \frac{d}{2}$$
$$I_{1\text{-}1} = \frac{d^4 - d_1^4}{12}$$
$$S_{1\text{-}1} = \frac{d^4 - d_1^4}{6d}$$
$$r_{1\text{-}1} = \sqrt{\frac{d^2 + d_1^2}{12}}$$

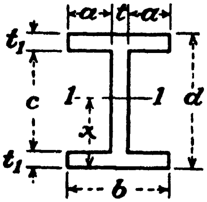


$$A = bd - b_1d_1$$
$$x = \frac{d}{2}$$
$$I_{1\text{-}1} = \frac{bd^3 - b_1d_1^3}{12}$$
$$S_{1\text{-}1} = \frac{bd^3 - b_1d_1^3}{6d}$$
$$r_{1\text{-}1} = \sqrt{\frac{bd^3 - b_1d_1^3}{12A}}$$

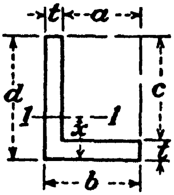
ELEMENTS OF SECTIONS



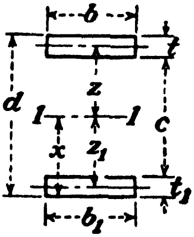
$$A = \frac{\pi(d^2 - d_1^2)}{4}$$
$$x = \frac{d}{2}$$
$$I_{1\text{-}1} = \frac{\pi(d^4 - d_1^4)}{64}$$
$$S_{1\text{-}1} = \frac{\pi(d^4 - d_1^4)}{32d}$$
$$r_{1\text{-}1} = \frac{\sqrt{d^2 + d_1^2}}{4}$$



$$A = bd - 2ac$$
$$x = \frac{d}{2}$$
$$I_{1\text{-}1} = \frac{bd^3 - 2ac^3}{12}$$
$$S_{1\text{-}1} = \frac{bd^3 - 2ac^3}{6d}$$
$$r_{1\text{-}1} = \sqrt{\frac{bd^3 - 2ac^3}{12A}}$$



$$A = t(a + d)$$
$$x = \frac{\frac{1}{2}d^2t + \frac{1}{2}t^2a}{A}$$
$$I_{1\text{-}1} = \frac{td^3 + at^3}{3} - Ax^2$$
$$S_{1\text{-}1} = \frac{I}{d - x}$$
$$r_{1\text{-}1} = \sqrt{\frac{I}{A}}$$



$$A = bt + b_1t_1$$
$$x = \frac{bt(d - \frac{1}{2}t) + \frac{1}{2}b_1t_1^2}{A}$$
$$I_{1\text{-}1} = \frac{bt^3}{12} + btz^2 + \frac{b_1t_1^3}{12} + b_1t_1z_1^2$$
$$S_{1\text{-}1} = \frac{I}{x}$$
$$r_{1\text{-}1} = \sqrt{\frac{I}{A}}$$

SQUARE AND ROUND BARS

Weights and Areas

Size, inches	Weight, pounds per foot		Area, square inches	
	□	○	□	○
0				
1/16	.013	.010	.0039	.0031
1/8	.053	.042	.0156	.0123
3/16	.120	.094	.0352	.0276
1/4	.213	.167	.0625	.0491
5/16	.332	.261	.0977	.0767
3/8	.478	.376	.1406	.1105
7/16	.651	.511	.1914	.1503
1/2	.850	.668	.2500	.1963
9/16	1.076	.845	.3164	.2485
5/8	1.328	1.043	.3906	.3068
11/16	1.607	1.262	.4727	.3712
3/4	1.913	1.502	.5625	.4418
13/16	2.245	1.763	.6602	.5185
7/8	2.603	2.044	.7656	.6013
15/16	2.988	2.347	.8789	.6903
1	3.400	2.670	1.0000	.7854
1/16	3.838	3.015	1.1289	.8866
1/8	4.303	3.380	1.2656	.9940
3/16	4.795	3.766	1.4102	1.1075
1/4	5.313	4.172	1.5625	1.2272
5/16	5.857	4.600	1.7227	1.3530
3/8	6.428	5.049	1.8906	1.4849
7/16	7.026	5.518	2.0664	1.6230
1/2	7.650	6.008	2.2500	1.7671
9/16	8.301	6.519	2.4414	1.9175
5/8	8.978	7.051	2.6406	2.0739
11/16	9.682	7.604	2.8477	2.2365
3/4	10.413	8.178	3.0625	2.4053
13/16	11.170	8.773	3.2852	2.5802
7/8	11.953	9.388	3.5156	2.7612
15/16	12.763	10.024	3.7539	2.9483
2	13.600	10.681	4.0000	3.1416
1/16	14.463	11.359	4.2539	3.3410
1/8	15.353	12.058	4.5156	3.5466
3/16	16.270	12.778	4.7852	3.7583
1/4	17.213	13.519	5.0625	3.9761
5/16	18.182	14.280	5.3477	4.2000
3/8	19.178	15.062	5.6406	4.4301
7/16	20.201	15.866	5.9414	4.6664
1/2	21.250	16.690	6.2500	4.9087
9/16	22.326	17.534	6.5664	5.1572
5/8	23.428	18.400	6.8906	5.4119
11/16	24.557	19.287	7.2227	5.6727
3/4	25.713	20.195	7.5625	5.9396
13/16	26.895	21.123	7.9102	6.2126
7/8	28.103	22.072	8.2656	6.4918
15/16	29.338	23.042	8.6289	6.7771
3	30.60	24.03	9.000	7.069
1/16	31.89	25.05	9.379	7.366
1/8	33.20	26.08	9.766	7.670
3/16	34.54	27.13	10.160	7.980

Size, inches	Weight, pounds per foot		Area, square inches	
	□	○	□	○
3				
1/4	35.91	28.21	10.563	8.296
5/16	37.31	29.30	10.973	8.618
3/8	38.73	30.42	11.391	8.946
7/16	40.18	31.55	11.816	9.281
1/2	41.65	32.71	12.250	9.621
9/16	43.15	33.89	12.691	9.968
5/8	44.68	35.09	13.141	10.321
11/16	46.23	36.31	13.598	10.680
3/4	47.81	37.55	14.063	11.045
13/16	49.42	38.81	14.535	11.416
7/8	51.05	40.10	15.016	11.793
15/16	52.71	41.40	15.504	12.177
4	54.40	42.73	16.000	12.566
1/16	56.11	44.07	16.504	12.962
1/8	57.85	45.44	17.016	13.364
3/16	59.62	46.83	17.535	13.772
1/4	61.41	48.23	18.063	14.186
5/16	63.23	49.66	18.598	14.607
3/8	65.08	51.11	19.141	15.033
7/16	66.95	52.58	19.691	15.466
1/2	68.85	54.07	20.250	15.904
9/16	70.78	55.59	20.816	16.349
5/8	72.73	57.12	21.391	16.800
11/16	74.71	58.67	21.973	17.257
3/4	76.71	60.25	22.563	17.721
13/16	78.74	61.85	23.160	18.190
7/8	80.80	63.46	23.766	18.665
15/16	82.89	65.10	24.379	19.147
5	85.00	66.76	25.000	19.635
1/16	87.14	68.44	25.629	20.129
1/8	89.30	70.14	26.266	20.629
3/16	91.49	71.86	26.910	21.135
1/4	93.71	73.60	27.563	21.648
5/16	95.96	75.36	28.223	22.166
3/8	98.23	77.15	28.891	22.691
7/16	100.53	78.95	29.566	23.221
1/2	102.85	80.78	30.250	23.758
9/16	105.20	82.62	30.941	24.301
5/8	107.58	84.49	31.641	24.850
11/16	109.98	86.38	32.348	25.406
3/4	112.41	88.29	33.063	25.967
13/16	114.87	90.22	33.785	26.535
7/8	117.35	92.17	34.516	27.109
15/16	119.86	94.14	35.254	26.688
6	122.40	96.13	36.000	28.274
1/16	124.96	98.15	36.754	28.866
1/8	127.55	100.18	37.516	29.465
3/16	130.17	102.23	38.285	30.069
1/4	132.81	104.31	39.063	30.680
5/16	135.48	106.41	39.848	31.296
3/8	138.18	108.53	40.641	31.919
7/16	140.90	110.66	41.441	32.548

SQUARE AND ROUND BARS
Weights and Areas

Size, inches	Weight, pounds per foot		Area, square inches		Size, inches	Weight, pounds per foot		Area, square inches	
	□	○	□	○		□	○	□	○
6					9				
1/2	143.65	112.82	42.250	33.183	1/4	290.91	228.48	85.563	67.201
9/16	146.43	115.00	43.066	33.824	5/16	294.86	231.58	86.723	68.112
5/8	149.23	117.20	43.891	34.472	3/8	298.83	234.70	87.891	69.029
11/16	152.06	119.43	44.723	35.125	7/16	302.83	237.84	89.066	69.953
3/4	154.91	121.67	45.563	35.785	1/2	306.85	241.00	90.250	70.882
13/16	157.79	123.93	46.410	36.450	9/16	310.90	244.18	91.441	71.818
7/8	160.70	126.22	47.266	37.122	5/8	314.98	247.38	92.641	72.760
15/16	163.64	128.52	48.129	37.800	11/16	319.08	250.61	93.848	73.708
7	166.60	130.85	49.000	38.485	3/4	323.21	253.85	95.063	74.662
1/16	169.59	133.19	49.879	39.175	13/16	327.37	257.12	96.285	75.622
1/8	172.60	135.56	50.766	39.871	7/8	331.55	260.40	97.516	76.589
3/16	175.64	137.95	51.660	40.574	15/16	335.76	263.71	98.754	77.561
1/4	178.71	140.36	52.563	41.282	10	340.00	267.04	100.000	78.540
5/16	181.81	142.79	53.473	41.997	1/16	344.26	270.38	101.254	79.525
3/8	184.93	145.24	54.391	42.718	1/8	348.55	273.75	102.516	80.516
7/16	188.07	147.71	55.316	43.445	3/16	352.87	277.14	103.785	81.513
1/2	191.25	150.21	56.250	44.179	1/4	357.21	280.55	105.063	82.516
9/16	194.45	152.72	57.191	44.918	5/16	361.58	283.99	106.348	83.525
5/8	197.68	155.26	58.141	45.664	3/8	365.98	287.44	107.641	84.541
11/16	200.93	157.81	59.098	46.415	7/16	370.40	290.91	108.941	85.563
3/4	204.21	160.39	60.063	47.173	1/2	374.85	294.41	110.250	86.590
13/16	207.52	162.99	61.035	47.937	9/16	379.33	297.92	111.566	87.624
7/8	210.85	165.60	62.016	48.707	5/8	383.83	301.46	112.891	88.664
15/16	214.21	168.24	63.004	49.483	11/16	388.36	305.02	114.223	89.710
8	217.60	170.90	64.000	50.265	3/4	392.91	308.59	115.563	90.763
1/16	221.01	173.58	65.004	51.054	13/16	397.49	312.19	116.910	91.821
1/8	224.45	176.29	66.016	51.849	7/8	402.10	315.81	118.266	92.886
3/16	227.92	179.01	67.035	52.649	15/16	406.71	319.45	119.629	93.957
1/4	231.41	181.75	68.063	53.456	11	411.40	323.11	121.000	95.033
5/16	234.93	184.52	69.098	54.269	1/16	416.03	326.80	122.379	96.116
3/8	238.48	187.30	70.141	55.088	1/8	420.80	330.50	123.766	97.205
7/16	242.05	190.11	71.191	55.914	3/16	425.54	334.22	125.160	98.301
1/2	245.65	192.93	72.250	56.745	1/4	430.31	337.97	126.563	99.402
9/16	249.28	195.78	73.316	57.583	5/16	435.11	341.73	127.973	100.510
5/8	252.93	198.65	74.391	58.426	3/8	439.93	345.52	129.391	101.623
11/16	256.61	201.54	75.473	59.276	7/16	444.78	349.33	130.816	102.743
3/4	260.31	204.45	76.563	60.132	1/2	449.65	353.16	132.250	103.869
13/16	264.04	207.38	77.660	60.994	9/16	454.55	357.00	133.691	105.001
7/8	267.80	210.33	78.766	61.863	5/8	459.48	360.87	135.141	106.139
15/16	271.59	213.31	79.879	62.737	11/16	464.43	364.76	136.598	107.284
9	275.40	216.30	81.000	63.617	3/4	469.41	368.68	138.063	108.434
1/16	279.24	219.31	82.129	64.504	13/16	474.42	372.61	139.535	109.591
1/8	283.10	222.35	83.266	65.397	7/8	479.45	376.56	141.016	110.754
3/16	286.99	225.41	84.410	66.296	15/16	484.51	380.54	142.504	111.923
					12	489.60	384.53	144.000	113.098

SCREW THREADS
American Standard Free Fit—Class 2

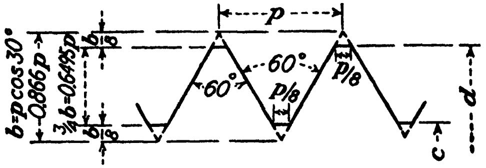


FIG. 119A.

Diameter		Area			Diameter		Area		
Total d, inches	Net c, inches	Total diameter, d, square inches	Net diameter, c, square inches	Number of threads per inch	Total d, inches	Net c, inches	Total diameter, d, square inches	Net diameter, c, square inches	Number of threads per inch
¼	.185	.049	.027	20	2½	2.175	4.909	3.716	4
⅜	.294	.110	.068	16	2¾	2.425	5.940	4.619	4
½	.400	.196	.126	13					
⅝	.507	.307	.202	11	3	2.629	7.069	5.428	3½
¾	.620	.442	.302	10	3¼	2.879	8.296	6.509	3½
⅞	.731	.601	.419	9	3½	3.100	9.621	7.549	3¼
					3¾	3.317	11.045	8.641	3
1	.838	.785	.551	8					
1⅛	.939	.994	.693	7	4	3.567	12.566	9.993	3
1¼	1.064	1.227	.890	7	4¼	3.798	14.186	11.330	2⅞
1⅜	1.158	1.485	1.054	6	4½	4.028	15.904	12.741	2¾
1½	1.283	1.767	1.294	6	4¾	4.255	17.721	14.221	2⅝
1⅝	1.389	2.074	1.515	5½					
1¾	1.490	2.405	1.744	5	5	4.480	19.635	15.766	2½
1⅞	1.615	2.761	2.049	5	5¼	4.730	21.648	17.574	2½
					5½	4.953	23.758	19.268	2⅜
2	1.711	3.142	2.300	4½	5¾	5.203	25.967	21.262	2⅜
2¼	1.961	3.976	3.021	4½					
					6	5.423	28.274	23.095	2¼

BOLT HEADS AND NUTS
United States and American Bridge Company Standard

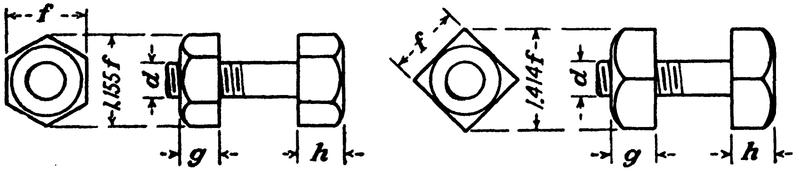


FIG. 119B.

Heads and nuts		U. S. Standard	A. B. Co. Standard
Head	Height h	$0.75d + \frac{1}{16}$ in.	$0.75d$
	Short diameter f	$1.50d + \frac{1}{8}$ in.	$1.50d$
Nut	Height g	d	d
	Short diameter f	$1.50d + \frac{1}{8}$ in.	$1.50d + \frac{1}{8}$ in.

Heads for Bolts 1½ in. and under, A. B. Co. Standard.
Heads for Bolts 1⅝ in. and over, U. S. Standard.

BOLT HEADS AND NUTS

American Bridge Company Standard

Diameter of bolt, inches	Head					Diameter of bolt, inches	Nut				
	Hexagon Diameter, inches		Height, inches	Square Diameter, inches			Hexagon Diameter, inches		Height, inches	Square Diameter, inches	
	Long	Short		Long	Short		Long	Short		Long	Short
1/4	7/16	3/8	3/16	1/2	3/8	1/4	9/16	1/2	1/4	11/16	1/2
3/8	5/8	9/16	1/4	3/4	9/16	3/8	13/16	11/16	3/8	1	11/16
1/2	7/8	3/4	3/8	1 1/16	3/4	1/2	1	7/8	1/2	1 1/4	7/8
5/8	1 1/8	15/16	1/2	1 3/8	15/16	5/8	1 1/4	1 1/16	5/8	1 1/2	1 1/16
3/4	1 1/4	1 1/8	9/16	1 5/8	1 1/8	3/4	1 1/2	1 1/4	3/4	1 3/4	1 1/4
7/8	1 1/2	1 5/16	5/8	1 7/8	1 5/16	7/8	1 5/8	1 7/16	7/8	2	1 7/16
1	1 3/4	1 1/2	3/4	2 1/8	1 1/2	1	1 7/8	1 5/8	1	2 1/4	1 5/8
1 1/8	2	1 11/16	7/8	2 3/8	1 11/16	1 1/8	2 1/8	1 13/16	1 1/8	2 5/8	1 13/16
1 1/4	2 1/8	1 7/8	15/16	2 5/8	1 7/8	1 1/4	2 1/4	2	1 1/4	2 7/8	2
1 3/8	2 3/8	2 1/16	1	2 7/8	2 1/16	1 3/8	2 1/2	2 3/16	1 3/8	3 1/8	2 3/16
1 1/2	2 5/8	2 1/4	1 1/8	3 1/8	2 1/4	1 1/2	2 3/4	2 3/8	1 1/2	3 3/8	2 3/8
1 5/8	3	2 9/16	1 1/4	3 5/8	2 9/16	1 5/8	3	2 9/16	1 5/8	3 5/8	2 9/16
1 3/4	3 1/8	2 3/4	1 3/8	3 7/8	2 3/4	1 3/4	3 1/8	2 3/4	1 3/4	3 7/8	2 3/4
1 7/8	3 3/8	2 15/16	1 1/2	4 1/8	2 15/16	1 7/8	3 3/8	2 15/16	1 7/8	4 1/8	2 15/16
2	3 5/8	3 1/8	1 9/16	4 3/8	3 1/8	2	3 5/8	3 1/8	2	4 3/8	3 1/8
2 1/4	4	3 1/2	1 3/4	5	3 1/2	2 1/4	4	3 1/2	2 1/4	5	3 1/2
2 1/2	4 1/2	3 7/8	1 15/16	5 1/2	3 7/8	2 1/2	4 1/2	3 7/8	2 1/2	5 1/2	3 7/8
2 3/4	4 7/8	4 1/4	2 1/8	6	4 1/4	2 3/4	4 7/8	4 1/4	2 3/4	6	4 1/4
3	5 3/8	4 5/8	2 5/16	6 1/2	4 5/8	3	5 3/8	4 5/8	3	6 1/2	4 5/8
3 1/4	5 3/4	5	2 1/2	7	5	3 1/4	5 3/4	5	3 1/4	7	5
3 1/2	6 1/4	5 3/8	2 11/16	7 5/8	5 3/8	3 1/2	6 1/4	5 3/8	3 1/2	7 5/8	5 3/8
3 3/4	6 5/8	5 3/4	2 7/8	8 1/8	5 3/4	3 3/4	6 5/8	5 3/4	3 3/4	8 1/8	5 3/4
4	7	6 1/8	3 1/16	8 5/8	6 1/8	4	7	6 1/8	4	8 5/8	6 1/8
4 1/4	7 1/2	6 1/2	3 1/4	9 1/4	6 1/2	4 1/4	7 1/2	6 1/2	4 1/4	9 1/4	6 1/2
4 1/2	8	6 7/8	3 7/16	9 3/4	6 7/8	4 1/2	8	6 7/8	4 1/2	9 3/4	6 7/8
4 3/4	8 3/8	7 1/4	3 5/8	10 1/4	7 1/4	4 3/4	8 3/8	7 1/4	4 3/4	10 1/4	7 1/4
5	8 3/4	7 5/8	3 13/16	10 3/4	7 5/8	5	8 3/4	7 5/8	5	10 3/4	7 5/8
5 1/4	9 1/4	8	4	11 1/4	8	5 1/4	9 1/4	8	5 1/4	11 1/4	8
5 1/2	9 5/8	8 3/8	4 3/16	11 7/8	8 3/8	5 1/2	9 5/8	8 3/8	5 1/2	11 7/8	8 3/8
5 3/4	10 1/8	8 3/4	4 3/8	12 3/8	8 3/4	5 3/4	10 1/8	8 3/4	5 3/4	12 3/8	8 3/4
6	10 1/2	9 1/8	4 9/16	12 7/8	9 1/8	6	10 1/2	9 1/8	6	12 7/8	9 1/8

LENGTH OF BOLT THREADS

Length of bolt, inches	Diameter of bolt, inches								
	1/4	3/8	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4
1 to 1 1/2	3/4	3/4	1	1 1/4					
1 5/8 to 2	3/4	3/4	1	1 1/4	1 1/2	1 5/8			
2 1/8 to 2 1/2	3/4	3/4	1	1 1/4	1 1/2	1 3/4	1 3/4		
2 5/8 to 3	7/8	7/8	1	1 1/4	1 1/2	1 3/4	1 3/4	2 1/4	
3 1/8 to 4	7/8	7/8	1 1/4	1 1/4	1 1/2	1 3/4	1 3/4	2 1/4	2 1/2
4 1/8 to 8	1	1	1 1/4	1 1/2	1 3/4	2	2 1/4	2 1/4	2 3/4
8 1/8 to 12	1	1	1 1/2	1 3/4	2	2 1/4	2 1/2	3	3
12 1/8 to 20	1	1	1 1/2	2	2	2 1/4	2 1/2	3	3

Bolts are usually threaded about 3 times the diameter; in no case are standard bolts threaded closer to the head than 1/8 in.

STRESSES PER SQUARE INCH

Metals and alloys	Stresses in kips					Modulus of elasticity, pounds	Elongation, %
	Tension, ultimate	Elastic limit	Compression ultimate	Bending, ultimate	Shearing, ultimate		
Aluminum: Cast	15	65	12	......	12	11,000,000	
Bars, sheets	24–28	12–14					
Wire, hard	30–65	16–30					
Wire, annealed	20–35	14					
2 % to 7 % Ni, Cu, Fe, etc	40–50	25					
Aluminum bronze: 5 % to 7½ % Al	75	40	120				
10 % Al	85–100	60					
Copper: Cast	25	6	40	22	30	10,000,000	
Plates, rods, bolts	32–35	10	32				
Wire, hard	55–65	....	......	......	....	18,000,000	
Wire, annealed	36	10	......	......	....	15,000,000	
Brass: 17 % Zn	32.6	8.2	......	23.2	....	....	26.7
23 % Zn	......	7.6	42	22.3	....	....	35.8
30 % Zn	28.1	8.6	......	26.9	....	....	20.7
39 % Zn	41.1	17.4	75	39	....	....	20.7
50 % Zn	31	17.9	117	33.5	....	....	5.0
Cast, common	18–24	6	30	20	36	9,000,000	
Wire, hard	80						
Wire, annealed	50	16	......	....	....	14,000,000	
Bronze: 8 % Sn	28.5	19	42	43.7	....	10,000,000	5.5
13 % Sn	29.4	20	53	34.5	....	....	3.3
20 % Sn	33	....	78	56.7	....	....	0.04
24 % Sn	22	22	114	32	....	....	0
30 % Sn	5.6	5.6	147	12.1	....	....	0
Gun metal, 9 Cu, 1 Sn	25–55	10	......	52	....	10,000,000	
Manganese, cast 10 % Sn	60	30	125				
Manganese, rolled 2 % Mn	100	80					
Bronze: Phosphorus, cast 9 % Sn	50	24					
Phosphorus, wire 1 % P	100						
Silicon cast, 3 % Si	55						
Silicon cast, 5 % Si	75						
Silicon wire	108						
Tobin, cast 38 % Zn	66						
Tobin, rolled 1½ % Sn	80	40	......	......	....	4,500,000	
Tobin, cold rolled ½ % Pb	100						
Lead: cast	1.8	....	......	......	....	1,000.000	
Pipe wire	2.2–2.5	....	......	......	....	1,000,000	
Rolled sheets	3.3	....	......	......	....	720,000	
Platinum: Wire unannealed	53						
Wire annealed	32						
Silver, cast	40						
Tin: Cast	3.5–4.6	1.5–1.8	6	4	....	4,000,000	
Antimony, 10Sn, 1Sb	11						
Zinc: Cast	4–6	4	18	7	....	13,000,000	
Rolled sheets	7–16						

STRESSES PER SQUARE INCH

Metals and alloys	Stresses in kips					Modulus of elasticity, pounds	Elongation, %
	Tension, ultimate	Elastic limit	Compression ultimate	Bending, ultimate	Shearing, ultimate		
Steel							
Shapes: Plates, bars							
Bridges	55-65	½ tensile	Tensile	Tensile	¾ tensile	29,000,000	27.3-23.1
Buildings	55-65	½ tensile	Tensile	Tensile	¾ tensile	29,000,000	25.5-21.5
Cars	50-65	½ tensile	Tensile	Tensile	¾ tensile	29,000,000	30.0-23.1
Locomotives	55-65	½ tensile	Tensile	Tensile	¾ tensile	29,000,000	27.3-23.1
Ships	60-72	½ tensile	Tensile	Tensile	¾ tensile	29,000,000	25.0-21.1
Boiler plates: Flange plates	55-65	½ tensile	Tensile	Tensile	¾ tensile	29,000,000	27.3-23.1
Fire box	52-62	½ tensile	Tensile	Tensile	¾ tensile	29,000,000	28.8-24.2
Rivets: Boilers	45-55	½ tensile	Tensile	Tensile	¾ tensile	29,000,000	33.3-27.3
Bridges	52-62	½ tensile	Tensile	Tensile	¾ tensile	29,000,000	28.8-24.2
Buildings	52-62	½ tensile	Tensile	Tensile	¾ tensile	29,000,000	28.8-24.2
Cars	52-62	½ tensile	Tensile	Tensile	¾ tensile	29,000,000	28.8-24.2
Ships	55-65	½ tensile	Tensile	Tensile	¾ tensile	29,000,000	27.3-23.0
Concrete bars: Plain, structural grade	55-70	33	Tensile	Tensile	¾ tensile	29,000,000	25.5-20.0
Plain, intermediate	70-90	40	Tensile	Tensile	¾ tensile	29,000,000	18.6-14.3
Plain, hard	80	50	Tensile	Tensile	¾ tensile	29,000,000	15.0
Deformed, structural grade	55-70	33	Tensile	Tensile	¾ tensile	29,000,000	22.7-17.9
Deformed intermediate	70-90	40	Tensile	Tensile	¾ tensile	29,000,000	16.1-11.3
Deformed, hard	80	50	Tensile	Tensile	¾ tensile	29,000,000	12.5
Cold twisted	......	55	Tensile	Tensile	¾ tensile	29,000,000	5.0
Castings: Soft	60	30	Tensile	Tensile	¾ tensile	29,000,000	26
Medium	70	38	Tensile	Tensile	¾ tensile	29,000,000	24
Hard	80	43	Tensile	Tensile	¾ tensile	29,000,000	17
Steel Alloys							
Nickel steel, 3.25% Ni: Shapes, plates, bars	85-100	50	Tensile	Tensile	¾ tensile	29,000,000	17.6-15.0
Rivets	70-80	45	Tensile	Tensile	¾ tensile	29,000,000	21.4-18.8
Eye bars, unannealed	95-110	55	Tensile	Tensile	¾ tensile	29,000,000	15.8-13.6
Eye bars, annealed	90-105	52	Tensile	Tensile	¾ tensile	29,000,000	20.0
Steel Springs and Wire							
Springs, untempered	65-110	40-70					
Wire: Unallealed	120	60					
Annealed	80	40					
Bridge cable	200	95					
Wrought Iron							
Shapes	48	26	Tensile	Tensile	⅚ tensile	28,000,000	
Bars	50	27	Tensile	Tensile	⅚ tensile	28,000,000	
Wire: Unannealed	80	....	......	......	....	15,000,000	
Annealed	60	27	......	......	....	25,000,000	
Cast Iron							
Common	15-18	6	80	30	18-20	12,000,000	
Gray	18-24	....	......	25-33			
Malleable	27-35	15-20	46	30	40		

STRESSES IN POUNDS PER SQUARE INCH

Building materials	Ultimate average stresses			Modulus of elasticity	Safe working stresses		
	Compression	Tension	Bonding		Compression	Bearing	Shearing
Stone							
Granite, gneiss, bluestone	12,000	1,200	1,600	7,000,000	1,200	1,200	200
Limestone, marble	8,000	800	1,500	7,000,000	800	800	150
Sandstone	5,000	150	1,200	3,000,000	500	500	150
Slate	10,000	3,000	5,000	14,000,000	1,000	1,000	175
Masonry							
Granite	......	.....	.....	....	420	600	
Limestone, bluestone	......	.....	.....	....	350	500	
Sandstone	......	.....	.....	....	280	400	
Rubble	......	.....	.....	....	140	250	
Coursed	......	.....	.....	....	170	250	
Brick: Medium burned	10,000	.....	.....	....	170	300	
Hard burned	15,000	.....	.....	....	210	300	
Pressed, paving brick	6,000						
Terra cotta	5,000						
Cement, Portland							
Neat, 28 days	7,040	740					
Neat, 90 days	7,350	740					
1:3 sand, 28 days	1,290	320					
1:3 sand, 90 days	1,490	340					

Stresses in Pounds per Square Inch

Concrete, P.C.	Compression
1:1:2	
Granite, trap rock	3,300
Furnace slag	3,000
Lime and sandstone, hard	3,000
Lime and sandstone, soft	2,200
Cinders	800
1:1½:3	
Granite, trap rock	2.800
Furnace slag	2,500
Lime and sandstone, hard	2,500
Lime and sandstone, soft	1,800
Cinders	700
1:2:4	
Granite, trap rock	2,200
Furnace slag	2,000
Lime and sandstone, hard	2.000
Lime and sandstone, soft	1.500
Cinders	600
1:2½:5	
Granite, trap rock	1.800
Furnace slag	1,600
Lime and sandstone, hard	1,600
Lime and sandstone, soft	1.200
Cinders	500
1:3:6	
Granite, trap rock	1,400
Furnace slag	1,300
Lime and sandstone, hard	1,300
Lime and sandstone, soft	1.000
Cinders	400

Reinforced Concrete

Modulus of elasticity	3,000,000	for ultimate compression over 2,900
	2.500.000	for ultimate compression up to 2,900
	2,000,000	for ultimate compression up to 2,200
	750,000	for ultimate compression under 800

LINEAR COEFFICIENTS OF EXPANSION FOR 1 DEGREE

Substance	Coefficient, n	
	Centigrade	Fahrenheit
Metals and alloys		
Aluminum, wrought	.0000231	.0000128
Brass	.0000188	.0000104
Bronze wire	.0000193	.0000107
Bronze	.0000181	.0000101
Copper	.0000168	.0000093
German silver	.0000183	.0000102
Gold	.0000150	.0000083
Iron, cast, gray	.0000106	.0000059
Iron, wrought	.0000120	.0000067
Iron wire	.0000124	.0000069
Lead	.0000286	.0000159
Nickel	.0000126	.0000070
Platinum	.0000090	.0000050
Platinum-iridium, 15% Ir	.0000081	.0000045
Silver	.0000192	.0000107
Steel, cast	.0000110	.0000061
Steel, hard	.0000132	.0000073
Steel, medium	.0000120	.0000067
Steel, soft	.0000110	.0000061
Tin	.0000210	.0000117
Zinc, rolled	.0000311	.0000173

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